CHAPTER 13

Geometry

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

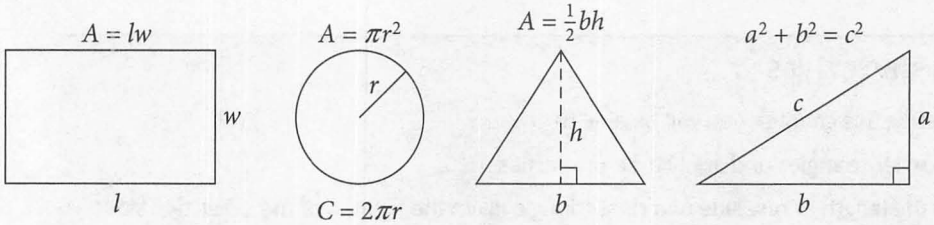
- Identify similar triangles and apply their properties
- Calculate the length of one side of a right triangle given the lengths of the other two sides
- Recognize the most common Pythagorean triples
- Calculate the other two sides of a 45-45-90 or 30-60-90 triangle, given one side length
- Interpret and manipulate the equation for a circle
- Calculate the length of an arc or the area of a sector defined by a central angle
- Convert degrees to radians
- Calculate the volume of common solids
- Calculate the surface area of common solids

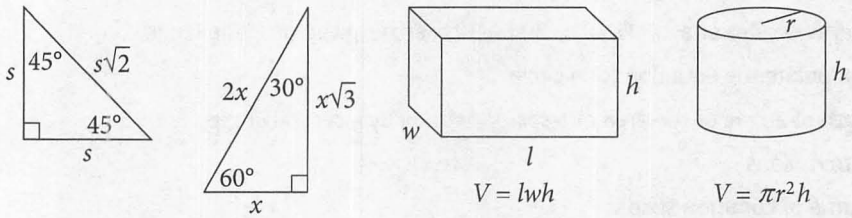
65/600 SmartPoints® (High Yield)

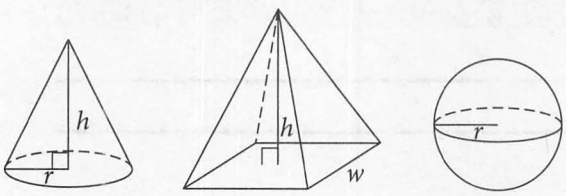
How Much Do You Know?

Directions: Try the questions that follow. Show your work so that you can compare your solutions to the ones found in the Check Your Work section immediately following this question set. The "Category" heading in the explanation for each question gives the title of the lesson that covers how to solve it. If you answered the question(s) for a given lesson correctly, and if your scratchwork looks like ours, you may be able to move quickly through that lesson. If you answered incorrectly or used a different approach, you may want to take your time on that lesson.

Make use of the formula sheet below as needed; you'll have these same formulas available in your test booklet when you take the real SAT.

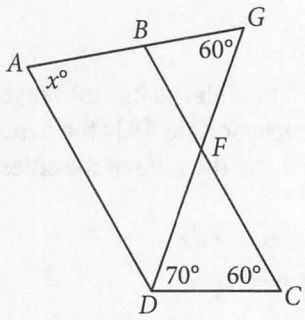






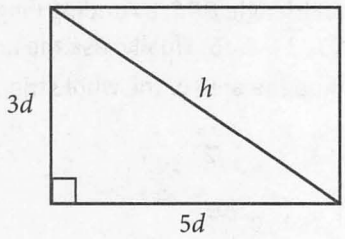
sum of interior angles of a triangle: 180°

full circle arc measure: $360° = 2\pi$ radians



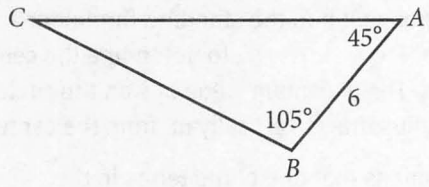
1. In the figure above, line segments AD and BC are parallel. What is the value of x?

   A) 60
   B) 70
   C) 80
   D) 110



2. In the figure above, the diagonal of the rectangle has length h. What is the value of h in terms of d?

   A) d
   B) $4d$
   C) $\sqrt{34d}$
   D) $d\sqrt{34}$

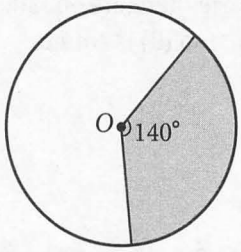


3. What is the area of the triangle shown in the figure?

   A) $18\sqrt{3}$
   B) $9+9\sqrt{3}$
   C) $9+18\sqrt{3}$
   D) $18+18\sqrt{3}$

4. What is the maximum value of y on the circumference of the circle defined by the equation $x^2+y^2+6x-10y-47=0$?

   A) 9
   B) 14
   C) 16
   D) 19



5. If the area of the shaded sector in circle O is 14π square units, what is the radius of the circle?

   A) 6
   B) 8
   C) 9
   D) 12

6. A yogurt factory fills cylindrical containers 80 percent of the way to the top, putting 6 ounces of yogurt in each cup. The containers are 4 inches tall and 2.5 inches wide. What is the approximate volume of 1 ounce of yogurt in cubic inches?

   A) 2.1
   B) 2.6
   C) 3.3
   D) 4.2

Check Your Work

1. B

Difficulty: Medium

Category: Similar Triangles

Getting to the Answer: Notice that $\triangle CDF$ and $\triangle BFG$ are similar: they share a 60° angle, and the vertical angles formed at point F are equal, so their third angles must be equal as well. It follows that $\angle FBG = 70°$. Because line segments AD and BC are parallel, $\angle FBG$ and $\angle BAD$ are corresponding angles (and are therefore equal). You can now conclude that $\angle x = 70°$, so **(B)** is correct.

2. D

Difficulty: Medium

Category: Pythagorean Theorem

Getting to the Answer: The sides $3d$, $5d$, and h form a right triangle, so plug these values into the Pythagorean theorem and then solve for h. When you square $3d$ and $5d$, be sure to square the coefficient and the variable:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (3d)^2 + (5d)^2 &= h^2 \\ 9d^2 + 25d^2 &= h^2 \\ 34d^2 &= h^2 \\ \sqrt{34d^2} &= h \\ d\sqrt{34} &= h \end{aligned}$$

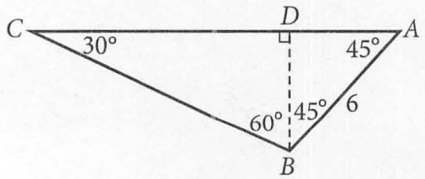
Choice **(D)** is correct.

3. B

Difficulty: Hard

Category: Special Right Triangles

Getting to the Answer: First, find the measure of the missing angle in the triangle: $180° - 105° - 45° = 30°$. Next, draw the height of the triangle up from B to a point, D, on side AC to create two right triangles:



Triangle ABD is a 45-45-90 triangle, so its side lengths are in the ratio $x{:}x{:}x\sqrt{2}$. Because side AB is the hypotenuse, set up an equation using the ratio of the sides:

$$\begin{aligned} 6 &= x\sqrt{2} \\ \frac{6}{\sqrt{2}} &= x \\ \frac{6}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) &= x \\ \frac{6\sqrt{2}}{2} &= x \\ 3\sqrt{2} &= x \end{aligned}$$

This is the length of both AD and BD. Triangle BDC is a 30-60-90 triangle, so its side lengths are in the ratio of $x{:}x\sqrt{3}{:}2x$. You just found that the length of the shorter leg, BD, is $3\sqrt{2}$, so multiply it by $\sqrt{3}$ to find the length of the longer leg $\left(\sqrt{3} \times 3\sqrt{2} = 3\sqrt{6}\right)$. Now, find the length of AC, which is the base of triangle BDC, by adding lengths AD to DC. The result is $3\sqrt{2} + 3\sqrt{6}$. Finally, use the area formula, $A = \frac{1}{2}bh$, to find the area of the whole triangle:

$$\begin{aligned} A \quad & \tfrac{1}{2}\, 3\sqrt{2} \quad 3\sqrt{6} \quad 3\sqrt{2} \\ & \tfrac{1}{2}\, 18 \quad 9\sqrt{12} \\ & \tfrac{1}{2}\, 18 \quad 9 \quad 2\sqrt{3} \\ & \tfrac{1}{2}\, 18 \quad 18\sqrt{3} \\ & 9 \quad 9\sqrt{3} \end{aligned}$$

Choice **(B)** is correct.

4. B

Difficulty: Medium

Getting to the Answer: Complete the square of the given equation to arrange it into the standard format for a circle, $(x-h)^2 + (y-k)^2 = r^2$, to determine the center and the radius. The maximum value of y on the circle will be that of a radius drawn vertically up from the center.

The value $6x$ means that one of the terms in the equation for the circle will be $(x+3)^2$ because that expands to $x^2 + 6x + 9$. Similarly, $10y$ means that the other term on the left side of the circle equation will be $(y-5)^2 = y^2 + 10y + 25$.

Rewrite the given equation by adding $9 + 25$ to each side: $x^2 + 6x + 9 + y^2 - 10y + 25 - 47 = 9 + 25$. Next, simplify the x and y terms and add 47 to each side to yield $(x + 3)^2 + (y - 5)^2 = 9 + 25 + 47 = 81$. So, the y-coordinate of the center is 5. Since $r^2 = 81$, the radius of the circle is 9. Therefore, the y-coordinate of the point straight above the center is $5 + 9 = 14$. **(B)** is correct.

5. A

Difficulty: Medium

Category: Arc Length and Sectors

Getting to the Answer: The question asks for the radius. If you know the area of the circle, you can find the radius. The question gives enough information to find the area of the circle. Use the relationship $\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{central angle}}{360^\circ}$:

$$\frac{14\pi}{A} = \frac{140}{360}$$
$$5{,}040\pi = 140A$$
$$36\pi = A$$

Now, solve for r using $A = \pi r^2$:

$$36\pi = \pi r^2$$
$$36 = r^2$$
$$\pm 6 = r$$

The radius can't be negative, so the correct answer is 6, which is **(A)**.

6. B

Difficulty: Hard

Category: Three-Dimensional Figures

Getting to the Answer: Find the volume of the container using the formula for volume of a cylinder, $V = \pi r^2 h$. The question gives you the width, or the diameter, of the container, so divide by 2 to get the radius: one-half of 2.5 is 1.25. Now, solve:

$$V = \pi(1.25)^2(4)$$
$$V = \pi(1.5625)(4)$$
$$V = 6.25\pi$$

The factory fills the cup only 80% of the way up, so multiply the container volume by 0.8 to find that the actual volume of the yogurt is $6.25\pi \times 0.8 = 5\pi$, or about 15.708 cubic inches. Divide this by 6 ounces to determine that 1 ounce takes up approximately 2.6 cubic inches of space, which matches **(B)**.

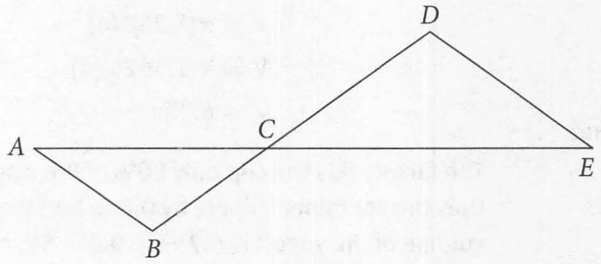
Similar Triangles

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Identify similar triangles and apply their properties

To answer a question like this:



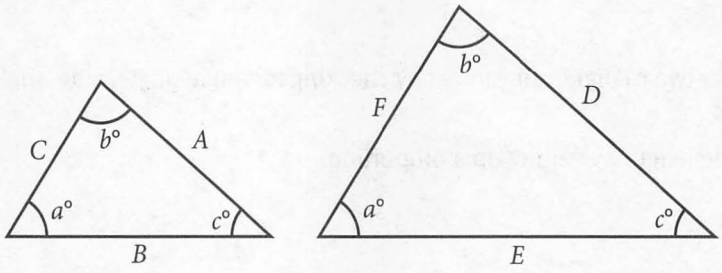
Note: Figure not drawn to scale.

In the figure above, segments AE and BD intersect at point C, and $\angle ABC \cong \angle CDE$. If $BC = DE = 5$ and $AB = 2$, what is the measure of CD?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

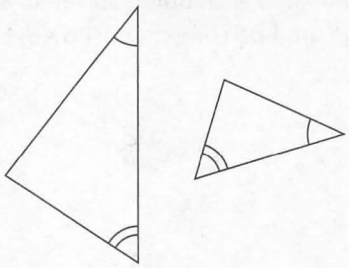
You need to know this:

The corresponding angles and side lengths of **congruent triangles** are equal. **Similar triangles** have the same angle measurements and proportional sides. In the figure that follows, the two triangles have the same angle measurements, so the side lengths can be set up as the following proportion: $\frac{A}{D} = \frac{B}{E} = \frac{C}{F}$.

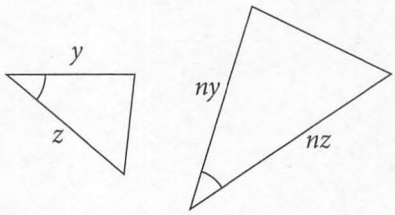


Two triangles are similar if three specific conditions are met:

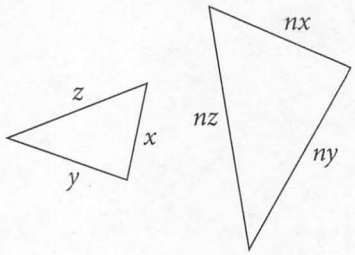
- Two of their three angles are congruent (**angle-angle**). For example, two triangles that each have one 40° and one 55° angle are similar.



- Two of their three sides are in the same proportion and the intervening angle is congruent (**side-angle-side**). For example, a triangle with sides of 10 and 12 and an intervening angle of 40° and another triangle with sides of 20 and 24 and an intervening angle of 40° are similar.



- Their three sides are in the same proportion (**side-side-side**). For example, a triangle with sides of 5, 6, and 8 and a triangle with sides of 15, 18, and 24 are similar.



You need to do this:

- Determine whether two triangles are similar by checking for angle-angle, side-angle-side, or side-side-side relationships.
- Find a missing side length by setting up a proportion.

Explanation:

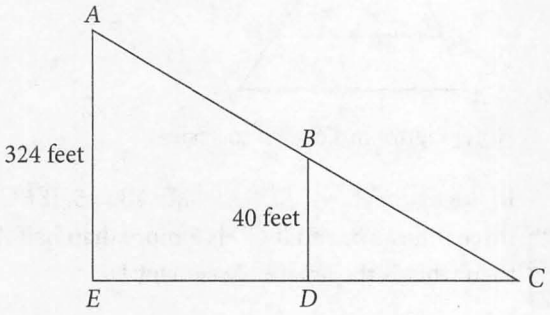
Label the figure with information from the question stem and information you can deduce from geometry principles. These two triangles are similar because they have two sets of congruent angles: one set is the set of vertical angles ($\angle ACB \cong \angle DCE$) and one set is given in the question stem ($\angle ABC \cong \angle CDE$). Corresponding sides in similar triangles are proportional to each other, so set up a proportion to find the missing side length. In this case, *CD* corresponds to *BC* and *DE* corresponds to *AB*. Let the measure of *CD* be represented by *x* and solve:

$$\frac{CD}{BC} = \frac{DE}{AB}$$
$$\frac{x}{5} = \frac{5}{2}$$
$$x = \frac{25}{2}$$

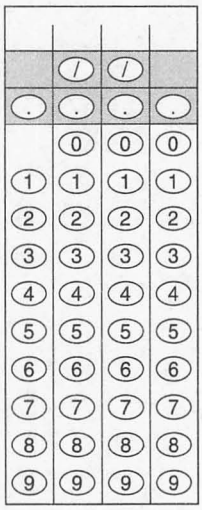
Grid in **25/2** or **12.5**.

Try on Your Own

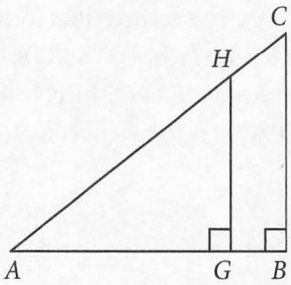
Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.



1. The diagram above shows similar triangles *ACE* and *BCD*. If segment *DC* is 50 percent longer than segment *BD*, how long is segment *DE*?



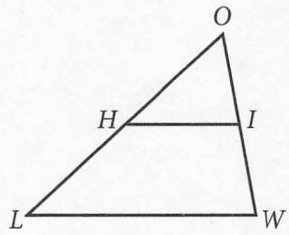
HINT: Triangles with a shared angle and parallel sides are similar. How can you use this fact in Q2?



Note: Figure not drawn to scale.

2. Triangle *ABC* above has an area of 150 square units. If lengths $AB = AH = 20$, then what is the length of *HG*?

   A) 5
   B) 12
   C) 16
   D) 20



Note: Figure not drawn to scale.

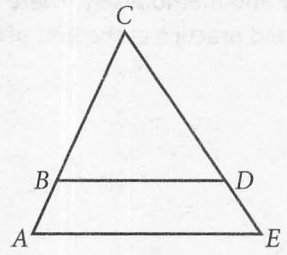
3. Triangle *LOW* is shown in the figure above, where segment *HI* is the bisector of both segments *LO* and *OW*. Given that $LW = 30$ and $HI = 4x - 1$, what is the value of x?

   A) 3.5
   B) 4
   C) 7.75
   D) 8

4. Right triangle DEF is similar to right triangle ABC, and both are plotted on a coordinate plane (not shown). The vertices of triangle DEF are $D(3, 2)$, $E(3, -1)$, and $F(-1, -1)$. The vertices that form triangle ABC's longer leg are $(-8, -3)$ and $(8, -3)$. If vertex A is in the same quadrant of the coordinate plane as vertex D, what is the y-coordinate of vertex A ?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

HINT: For Q5, translate carefully from English into math and fill in the lengths on the figure.



Note: Figure not drawn to scale.

5. In the figure above, $\overline{BD} \| \overline{AE}$ and $AB = 5$. If BC is three times AB, and if CD is 2 more than half AC, then what is the length of segment DE ?

A) 3
B) 4
C) 5
D) 6

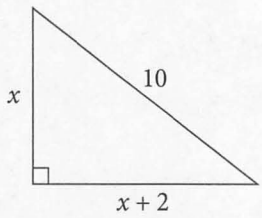
Pythagorean Theorem

LEARNING OBJECTIVES

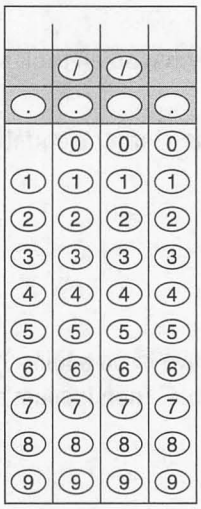
After this lesson, you will be able to:

- Calculate the length of one side of a right triangle given the lengths of the other two sides
- Recognize the most common Pythagorean triples

To answer a question like this:

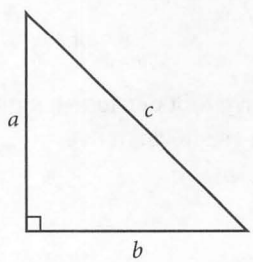


What is the area of the triangle shown?



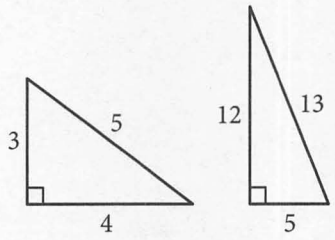
You need to know this:

The **Pythagorean theorem** states that in any right triangle (and *only* in right triangles), the square of the hypotenuse (the longest side) is equal to the sum of the squares of the legs (the shorter sides). If you know the lengths of any two sides of a right triangle, you can use the Pythagorean equation, $a^2 + b^2 = c^2$, to find the length of the third. In this equation, a and b are the legs of the triangle and c is the hypotenuse, the side across from the right angle of the triangle.



Consider an example: a right triangle has a leg of length 9 and a hypotenuse of length 14. To find the missing leg, plug the known values into the Pythagorean equation: $9^2 + b^2 = 14^2$. This simplifies to $81 + b^2 = 196$, which becomes $b^2 = 115$. Take the square root of both sides to find that $b = \sqrt{115}$.

Some right triangles have three side lengths that are all integers. These sets of integer side lengths are called **Pythagorean triples**. The two most common Pythagorean triples on the SAT are 3:4:5 and 5:12:13. Look for multiples of these (e.g., 6:8:10 and 10:24:26) as well. Memorizing these triples now can save you valuable calculation time on test day.



You need to do this:

- Keep in mind that the Pythagorean theorem applies only to right triangles.
- When you need to find a side length of a right triangle, look first for the common Pythagorean triples or their multiples.
- If you cannot identify any Pythagorean triples, substitute any two known side lengths into the equation $a^2 + b^2 = c^2$, where c represents the hypotenuse, to find the third.

Explanation:

You could answer this question by applying the Pythagorean equation, but you would have to do a fair amount of algebra. It's much faster if you recognize that this is a 6:8:10 Pythagorean triple. Then it's just a matter of calculating the area, using the legs as the base and height:

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24$$

For the record, here's the solution using the Pythagorean equation:

$$\begin{aligned} x^2 + (x+2)^2 &= 10^2 \\ x^2 + x^2 + 4x + 4 &= 100 \\ 2x^2 + 4x - 96 &= 0 \\ x^2 + 2x - 48 &= 0 \\ (x+8)(x-6) &= 0 \\ x = -8 \quad &\text{or} \quad x = 6 \end{aligned}$$

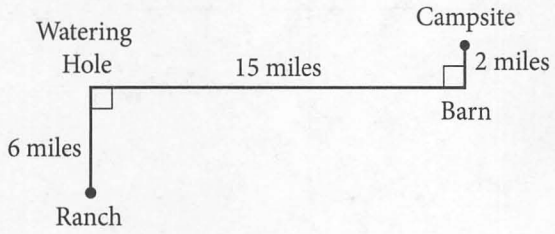
At this point, you would throw out the negative root because length must be positive, so you would use 6 and $6 + 2 = 8$ as the legs and calculate the area as shown above.

Grid in **24**.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

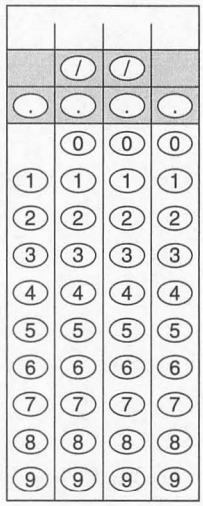
HINT: For Q6, draw the direct path and use it as the hypotenuse of a right triangle.



6. A tourist ranch built the horse-riding trail shown in the figure. The trail takes a rider from the ranch to an old watering hole, then to a historic barn, and finally to a campsite where riders can spend the night. If a rider took a horse on a direct path from the ranch to the campsite, how much shorter, in miles, would the trip be?

   A) 6
   B) 8
   C) 17
   D) 23

7. When Ted earned his driver's license, he wanted his first solo drive to be to a friend's house. Previously, Ted had always biked to his friend's house and was able to cut through the yards of neighbors to travel in a straight line. In his car, however, Ted travels a longer distance as he follows the streets. As a result, he travels 6 miles east, 6 miles south, and 2 more miles east by car. How much shorter, in miles, is Ted's bike route than his car route?



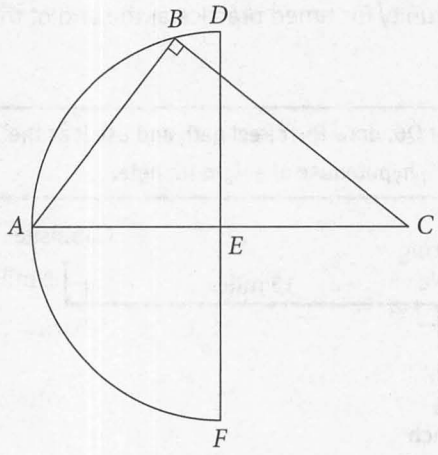
8. During a camping trip, Aundria and Annette decide to travel to their campsite using two different routes. Aundria takes the hiking trail that travels 5 miles south, 6 miles east, 7 miles south, and 2 miles west to the campsite; Annette uses the cross-country route that starts at the same point as the hiking trail but goes in a straight line from there to the campsite. About how many miles total will the two travel?

   A) 32.65
   B) 33.42
   C) 34.00
   D) 34.42

9. The lengths of the legs of a right triangle are $3x$ and $x + 1$. The hypotenuse is $3x + 1$. What is the value of x?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

HINT: For Q10, use the triangle to find the radius of the semicircle.



10. In the figure above, E is the center of the semicircle and $AE = EC$. If $AB = 5$ and $BC = 12$, then what is the area of the semicircle?

A) 13π

B) 26π

C) $\frac{169}{4}\pi$

D) $\frac{169}{8}\pi$

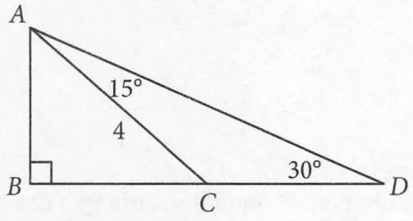
Special Right Triangles

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Calculate the other two sides of a 45-45-90 or 30-60-90 triangle, given one side length

To answer a question like this:



Given triangle *ABC* and triangle *ABD* above, what is the perimeter of triangle *ACD* ?

A) $2\sqrt{6} - 2\sqrt{2}$

B) $4\sqrt{3}$

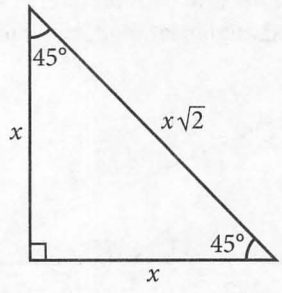
C) $4 + 2\sqrt{6} + 2\sqrt{2}$

D) $2\sqrt{6} + 6\sqrt{2}$

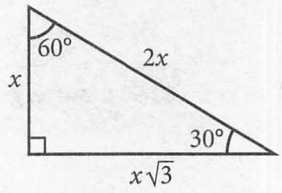
You need to know this:

The question above requires you to be able to recognize two **special right triangles**. These triangles are defined by their angles. As a result, the ratios of their side lengths are always the same. If you know the length of any one of the three sides of a special right triangle, you can find the lengths of the other two.

The ratio of the sides of a **45-45-90** triangle is $x{:}x{:}x\sqrt{2}$, where x is the length of each leg and $x\sqrt{2}$ is the length of the hypotenuse:



The ratio of the sides of a **30-60-90** triangle is $x{:}x\sqrt{3}{:}2x$, where x is the shorter leg, $x\sqrt{3}$ is the longer leg, and $2x$ is the hypotenuse:



These side length ratios are given on the SAT formula sheet, but for the sake of efficiency on test day, we recommend that you memorize them.

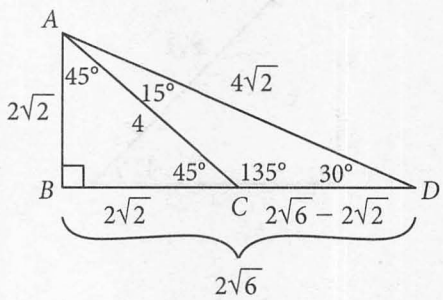
You need to do this:

- Look for hidden special right triangles within other shapes. For example, an equilateral triangle can be bisected (cut in half) to form two congruent 30-60-90 triangles and a square can be divided with a diagonal into two congruent 45-45-90 triangles.
- Use one known side length to deduce the other two. For example, if the shorter leg of a 30-60-90 triangle has a length of 5, then the longer leg has a length of $5\sqrt{3}$, and the hypotenuse has a length of $5(2) = 10$.

Explanation:

Look for hidden special right triangles and add new information to your diagram as you go. Start by finding $\angle ACD$ from the two given angles: $\angle ACD = 180° - 15° - 30° = 135°$. Because $\angle ACB$ is supplementary to $\angle ACD$, $\angle ACB$ measures 45°. ΔABC is a right triangle, so its missing angle ($\angle BAC$) is also 45°, making ΔABC a 45-45-90 triangle. Since $\angle ADB$ is 30° and $\angle ABC$ is 90°, $\angle BAD$ is 60° and ΔABD is a 30-60-90 triangle.

Knowing that you have two special right triangles will allow you to unlock the unknown side lengths. AC is the hypotenuse of the 45-45-90 triangle (side ratio of $x{:}x{:}x\sqrt{2}$), so AB and BC (the two legs) must be $2\sqrt{2}$ (solve the equation $4 = x\sqrt{2}$ to find this). AB is also the shorter leg of the 30-60-90 triangle (side ratio of $x{:}x\sqrt{3}{:}2x$), so BD (the longer leg) is $2\sqrt{6}$ and AD (the hypotenuse) is $4\sqrt{2}$. To determine CD, take the difference of BD and BC: $2\sqrt{6} - 2\sqrt{2}$. With all sides and angles labeled, the figure looks like this:



You now have all three sides of triangle ACD, so add them together for the perimeter:

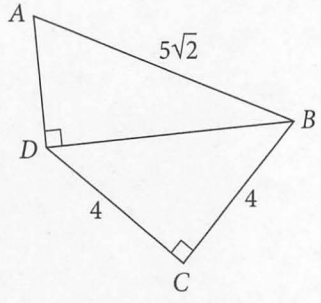
$$4 + \left(2\sqrt{6} - 2\sqrt{2}\right) + 4\sqrt{2}$$

This simplifies to $4 + 2\sqrt{6} + 2\sqrt{2}$, so **(C)** is correct.

Try on Your Own

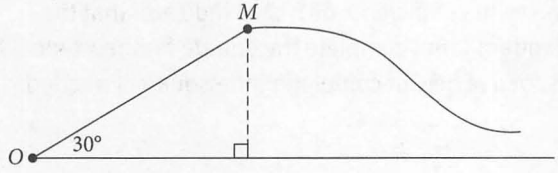
Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q11, don't rush to use the Pythagorean theorem. Which of the special triangles is triangle *ADB* ?



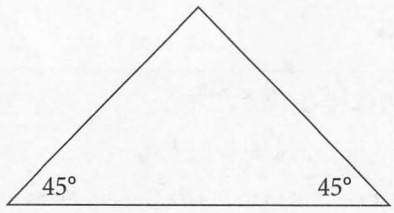
11. What is the area of triangle *DAB* ?

A) $3\sqrt{2}$
B) 8
C) 12
D) $20\sqrt{2}$



12. The distance from point *O* to point *M* of an amusement ride course shown in the figure above is $200\sqrt{3}$ feet. If the angle of ascent is 30°, what is the height, in feet, of the amusement ride at point *M* ?

A) $\frac{20}{3}\sqrt{3}$
B) $100\sqrt{3}$
C) 200
D) 300



13. Jonas plans to paint the top face of three corner shelves, one of which is shown above. If the longest side of one shelf is 10 inches, what is the total surface area, in square inches, Jonas needs to paint?

A) 25
B) 50
C) 75
D) 100

14. A tablet screen has a 12-inch diagonal. If the length of the screen is $\sqrt{3}$ times longer than the width, what is the area, in square inches, of the screen?

A) 12
B) $12\sqrt{3}$
C) 36
D) $36\sqrt{3}$

15. A theater is building a portable ramp to allow equipment and people easy access to the stage, which is 2 meters high. If the ramp is 4 meters long, what is the angle of the incline in degrees?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Circles

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Interpret and manipulate the equation for a circle

To answer a question like this:

Which of the following points in the *xy*-plane represents the center of the circle defined by the equation $x^2 + y^2 - 4x - 8y - 16 = 0$?

A) $(-4, -8)$

B) $(-2, -4)$

C) $(2, 4)$

D) $(4, 8)$

You need to know this:

The equation of a circle in the coordinate plane is as follows:

$$(x - h)^2 + (y - k)^2 = r^2$$

In this equation, called the **standard form**, r is the radius of the circle, and h and k are the x- and y-coordinates of the circle's center, respectively: (h, k).

You might also see what is referred to as **general form**:

$$x^2 + y^2 + Cx + Dy + E = 0$$

In the general form, the fact that there are x^2 and y^2 terms with coefficients of 1 is an indicator that the equation does indeed graph as a circle. To convert to standard form, complete the square for the x terms, then repeat for the y terms. Refer to chapter 12 on quadratics for a review of completing the square if needed.

You need to do this:

- Get the circle into standard form.
- Determine the center and radius using the standard form equation.
- Use the center and/or radius to answer the question.

Explanation:

The equation given is in general form rather than in standard form, so complete the square for both x and y. Start by grouping the x terms together and y terms together:

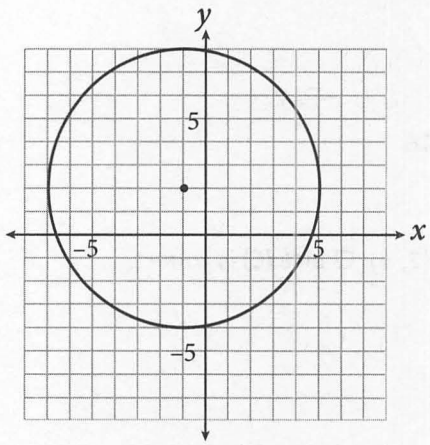
$$x^2 - 4x + y^2 - 8y = 16$$
$$(x - 4x + ?) + (y - 8y + ?) = 16$$
$$(x - 4x + 4) + (y - 8y + 16) = 16 + 4 + 16$$
$$(x - 2)^2 + (y - 4)^2 = 36$$

With the equation in standard form, it is now easy to see that the center is $(2, 4)$. Choice **(C)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q16, which part of the circle's equation would be the easiest to use to eliminate answer choices? The coordinates of the circle's center? The radius?



16. Which of the following represents the equation of the circle shown above?

A) $(x-1)^2+(y+2)^2=6$

B) $(x+1)^2+(y-2)^2=6$

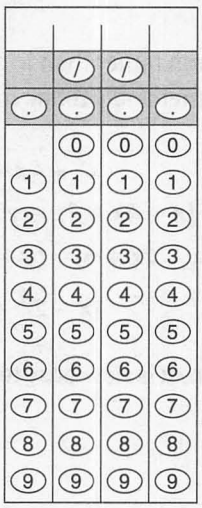
C) $(x-1)^2+(y+2)^2=36$

D) $(x+1)^2+(y-2)^2=36$

HINT: For Q17, complete the square for the x and y terms.

$x^2+6x+y^2-8y=171$

17. The equation of a circle in the xy-plane is shown above. What is the positive difference between the x- and y-coordinates of the center of the circle?



$x^2+y^2+8x-20y=28$

18. What is the diameter of the circle given by the equation above?

A) 12

B) 24

C) 28

D) 56

19. A circle in the xy-plane is defined by the equation $(x-4)^2+(y+2)^2=100$. Which of the following points is located on the circumference of the circle?

A) $(-3, 5)$

B) $(0, 9)$

C) $(4, -2)$

D) $(4, 8)$

Arc Length and Sectors

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Calculate the length of an arc or the area of a sector defined by a central angle
- Convert degrees to radians

To answer a question like this:

A circle with a diameter of 8 inches is divided into a number of equal sectors such that each sector has an area of $\frac{4\pi}{3}$ square inches. What is the central angle of each sector, in radians?

A) $\frac{\pi}{12}$

B) $\frac{\pi}{6}$

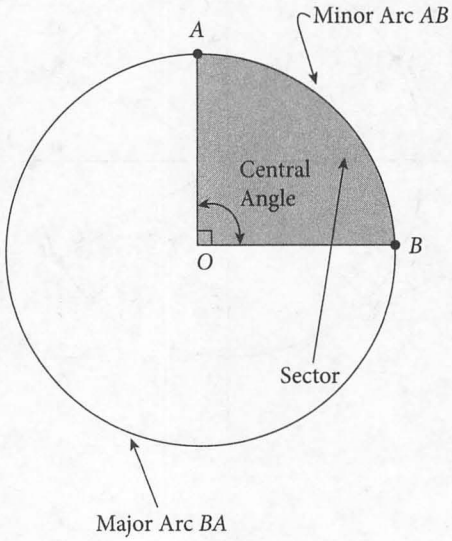
C) $\frac{\pi}{3}$

D) $\frac{2\pi}{3}$

You need to know this:

The SAT may ask you about the following parts of circles: arcs, central angles, and sectors. The ability to set up ratios and proportions correctly is essential for these questions.

- An **arc** is part of a circle's circumference. If the circumference is divided into exactly two arcs, the smaller one is called the **minor arc** and the larger one is called the **major arc**. If a diameter cuts the circle in half, the two arcs formed are called **semicircles**. An arc length can never be greater than the circle's circumference.
- An angle formed by two radii is called a **central angle**. Because a full circle contains 360°, a central angle measure cannot be greater than this.
- The part of a circle's area defined by a central angle is called a **sector**. The area of a sector cannot be greater than the circle's total area.

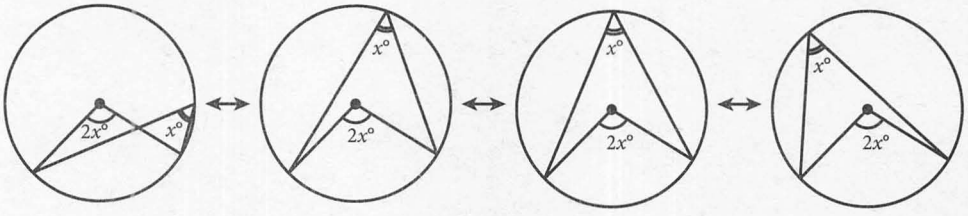


Here's a summary of the ratios formed by these three parts and their whole counterparts:

$$\frac{\text{central angle}}{360°} = \frac{\text{arc length}}{\text{circumference}} = \frac{\text{sector area}}{\text{circle area}}$$

Notice that all of these ratios are equal. Intuitively, this should make sense: when you slice a pizza into four equal slices, each piece should have $\frac{1}{4}$ of the cheese, crust, and sauce. If you slice a circle into four equal pieces, the same principle applies: each piece should have $\frac{1}{4}$ of the degrees, circumference, and area.

An angle whose vertex is on the edge of the circle is called an **inscribed angle**. As this vertex moves along the edge, the measure of the inscribed angle remains constant as long as the minor arc created does not change. When the line segments that create an inscribed angle define the same minor arc that a pair of radii do, a special relationship appears: the central angle measure is twice that of the inscribed angle.

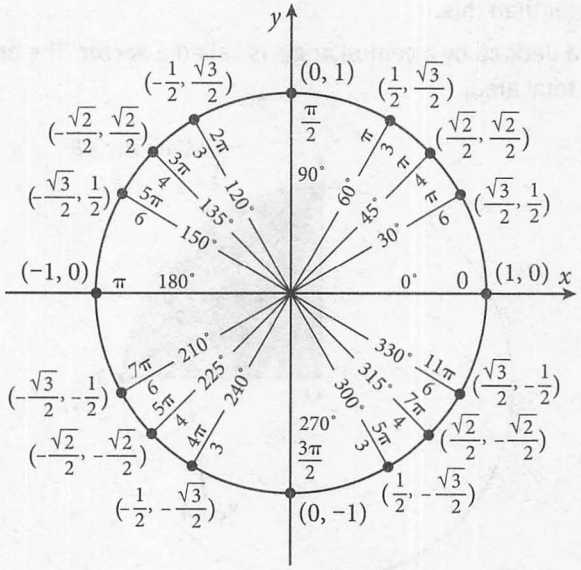


x remains constant

Most Geometry questions present angle measures in degrees, but some may present angle measures in radians. To convert between degrees and radians, use this relationship as a conversion factor: $180° = \pi$ radians. For instance, if you're asked to convert 90° into radians: $90° \times \frac{\pi}{180°} = \frac{\pi}{2}$. Note that there isn't a symbol for radians, so $\frac{\pi}{2}$ is read as "$\frac{\pi}{2}$ radians." This conversion works in the opposite direction as well: to convert radians to degrees, multiply by $\frac{180°}{\pi}$.

Most graphing calculators have both degree and radian modes, so make sure you're in the correct mode for the problem you're working on.

Below is a detailed unit circle diagram with common degree and radian measures (and the coordinates of the ends of their respective radii) that you may see on test day:



The coordinates at a particular angle measure translate into the leg lengths of the triangle created when a vertical line is drawn from the end of the radius down (or up if you're in quadrant III or IV) to the *x*-axis. For example, at 60°, the horizontal leg has a length of $\frac{1}{2}$ and the vertical leg has a length of $\frac{\sqrt{3}}{2}$.

You need to do this:

- To find the length of an arc or the area of a sector, you need to know the angle that defines the arc or sector as well as the radius of the circle. Questions that are especially tricky might not give you those values directly but will instead give you a way of calculating them.
- When converting between degrees and radians, set up the conversion fraction so that the units cancel:

$$\cancel{\text{degrees}} \times \frac{\pi \text{ radians}}{180 \cancel{\text{degrees}}} \quad \text{and} \quad \cancel{\text{radians}} \times \frac{180 \text{ degrees}}{\cancel{\text{radians}}}$$

Explanation:

Use the radius to find the area of the circle and divide the total area by the area of one sector to find the number of sectors. Divide the 360° in the whole circle by the number of sectors to find the central angle of each. Finally, convert to radians.

The question says the diameter of the circle is 8 inches, so the radius is 4 inches. A circle with a radius of 4 inches has an area of $\pi r^2 = 16\pi$. The area of each sector multiplied by the number of sectors will be equal to the total area:

$$\text{Area of a sector} \times \text{number of sectors} = \text{total area}$$

$$\text{Number of sectors} = \frac{\text{total area}}{\text{area of a sector}} = \frac{16\pi}{\frac{4\pi}{3}}$$

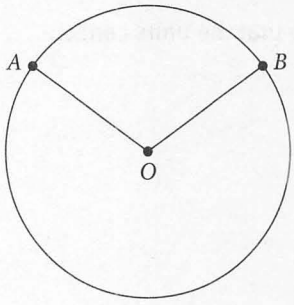
$$= 16\pi \times \frac{3}{4\pi} = 4 \times 3 = 12$$

If there are 12 sectors, the central angle defining each sector must be $\frac{1}{12}$ of 360°, or $\frac{1}{12} \times 2\pi = \frac{\pi}{6}$ radians. Choice **(B)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

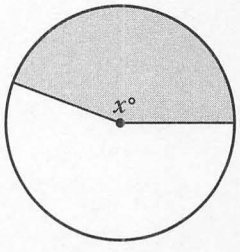
HINT: For Q20, what fraction of the total circumference is arc *AB* ? What does that tell you about central angle *AOB* ?



Note: Figure not drawn to scale.

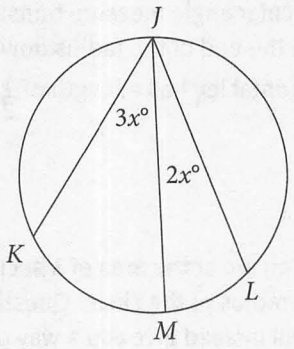
20. In the figure above, circle O has a radius of 120 centimeters. If the length of minor arc AOB is 200 centimeters, what is the measure of minor angle AOB, to the nearest tenth of a degree?

A) 95.5
B) 98.2
C) 102.1
D) 105.4



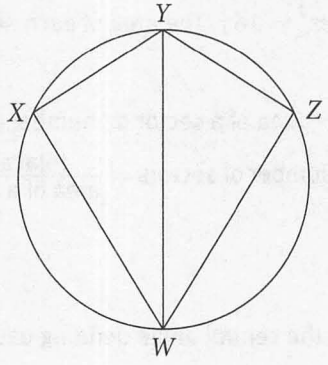
21. In the figure above, the ratio of the shaded area to the non-shaded area is 4 to 5. What is the value of x, in degrees?

A) 135
B) 145
C) 160
D) 170



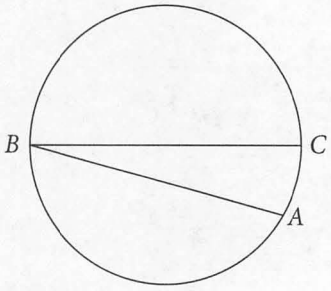
22. In the figure above, points J, K, L, and M lie on the circle. If the measure of arc KML is 150°, what is the value of x ?

A) 12
B) 15
C) 30
D) 60



23. In the figure above, points W, X, Y, and Z lie on the circle. The measure of arc YXW is 180°, and the measure of arcs XY and YZ each is 60°. If $YZ = 3$, then what is the length of WX ?

A) $\sqrt{3}$
B) 3
C) $3\sqrt{3}$
D) 6



24. In the figure above, BC is the diameter of the circle. If the measure of arc AC is 30°, then what is the measure of minor arc BA, in radians?

A) $\frac{\pi}{2}$

B) $\frac{2\pi}{3}$

C) $\frac{3\pi}{4}$

D) $\frac{5\pi}{6}$

Three-Dimensional Figures

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Calculate the volume of common solids
- Calculate the surface area of common solids

To answer a question like this:

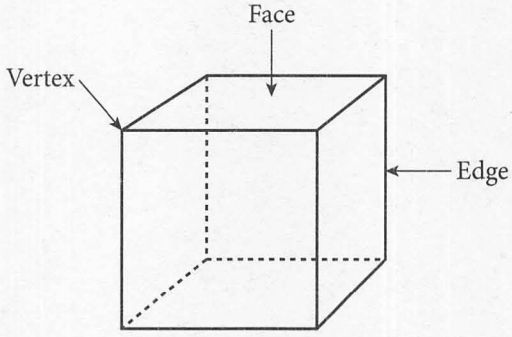
Marcus's yard has a large square sandbox serving as a cactus bed. The side lengths of the box are 24 feet, and the box is currently one-third full of sand from the bottom up. Marcus purchases 480 cubic feet of sand, which, when added to the box, will completely fill it. How many inches deep is the cactus bed?

A) 10
B) 15
C) 20
D) 25

You need to know this:

Over the last several sections, you learned about two-dimensional (2-D) shapes and how to tackle SAT questions involving them. Now you'll learn how to do the same for questions containing three-dimensional (3-D) shapes, also called solids. There are several different types of solids that might appear on the SAT—rectangular solids, cubes, cylinders, prisms, spheres, cones, pyramids—knowing their structures will help you on test day.

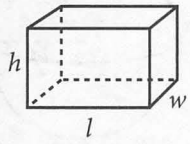
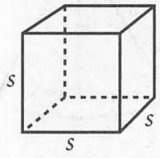
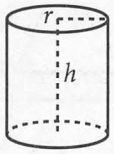
The following diagram shows the basic anatomy of a 3-D shape:



A **face** (or **surface**) is a 2-D shape that acts as one of the sides of the solid. Two faces meet at a line segment called an **edge**, and three faces meet at a single point called a **vertex**.

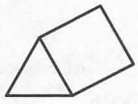
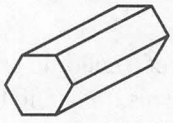
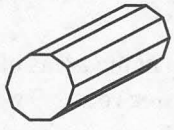
Volume

Volume is the amount of 3-D space occupied by a solid. Volume is analogous to the area of a 2-D shape. You can find the volume of many 3-D shapes by finding the area of the base and multiplying it by the height. In the table of formulas, the pieces that represent the areas of the bases are enclosed in parentheses.

Rectangular Solid	Cube	Right Cylinder
		
$(l \times w) \times h$	$(s \times s) \times s = s^3$	$(\pi \times r^2) \times h$

The three 3-D shapes shown above are prisms. Almost all prisms on the SAT are right prisms; that is, all faces are perpendicular to those with which they share edges.

Following are some examples of less commonly seen prisms:

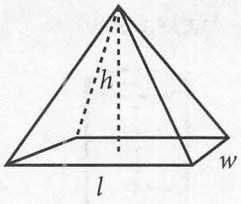
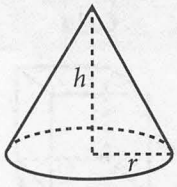
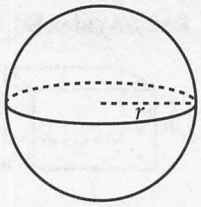
Triangular Prism	Hexagonal Prism	Decagonal Prism
		

Like the rectangular solids, cubes, and cylinders you saw earlier, these right prisms use the same general volume formula ($V = A_{base} \times h$).

You might not be told explicitly the area of the base of a prism, in which case you'll need to rely on your two-dimensional geometry knowledge to find it before calculating the volume.

More complicated 3-D shapes include the right pyramid, right cone, and sphere. The vertex of a right pyramid or right cone will always be centered above the middle of the base. Their volume formulas are similar to those of prisms, albeit with different coefficients.

Some of these formulas might look daunting, but you won't have to memorize them for test day. They'll be provided on the reference page at the beginning of each Math section.

Right Rectangular Pyramid	Right Cone	Sphere
		
$\frac{1}{3} \times (l \times w) \times h$	$\frac{1}{3} \times (\pi \times r^2) \times h$	$\frac{4}{3} \times \pi \times r^3$

A right pyramid can have any polygon as its base, but the square variety is the one you're most likely to see on the SAT. Also note that the vertex above the base of a right pyramid or cone is not necessarily formed by an intersection of exactly three faces, as in prisms, but it is still a single point and is still called a vertex.

Surface Area

Surface area is the sum of the areas of all faces of a solid. To calculate the surface area of a solid, simply find the area of each face using your 2-D geometry skills, then add them all together.

You won't be expected to know the surface area formulas for right pyramids, right cones, and spheres. They'll be provided at the beginning of each Math section. However, you could be asked to find the surface area of a prism, in which case you'll be given enough information to find the area of each surface of the solid.

You might think that finding the surface area of a solid with many sides, such as a right hexagonal prism, is a tall order. However, you can save time by noticing a vital trait: this prism has two identical hexagonal faces and six identical rectangular faces. Don't waste time finding the area of each of the eight surfaces. Find the area of one hexagonal face and one rectangular face only. Then multiply the area of the hexagonal face by 2 and the area of the rectangular face by 6, add the products together, and you're done. The same is true for other 3-D shapes such as rectangular solids (including cubes), other right prisms, and certain pyramids.

You need to do this:

- To answer questions that involve regular solids, look for ways to find the area of the base and the height.
- To answer questions that involve solids that are not regular, look up and apply the appropriate formula.
- To answer questions that involve surface area, look for surfaces that are the same. Calculate the area of each kind of surface once, and then multiply by the number of identical surfaces in the solid.

Explanation:

One way to approach this question is to find the total volume of the cactus bed first and then use that volume to calculate its depth. If the bed is one-third full, it is two-thirds empty, and according to the question, this empty space is 480 cubic feet. So two-thirds of the total volume is 480:

$$\frac{2}{3} \times V = 480 \text{ ft}^3$$

$$V = 480 \times \frac{3}{2} = 720 \text{ ft}^3$$

The volume of the cactus bed is also equal to the product of its length, width, and height. The question states that the length and width are each 24 feet, so plug these values into the formula and solve for height:

$$V = l \times w \times h$$

$$h = \frac{V}{l \times w}$$

$$= \frac{720}{24 \times 24}$$

$$= \frac{720}{576}$$

$$= 1.25 \text{ feet}$$

The question asks for inches, so do the conversion:

$$1.25 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} = 15 \text{ inches}$$

Choice **(B)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q25, refer to the formulas at the beginning of the chapter. On test day, they are included at the beginning of each Math section in your test booklet.

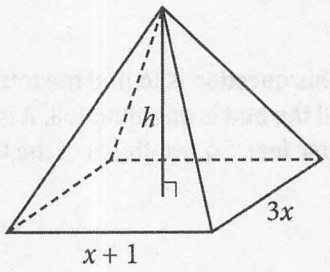
25. Two ornamental glass spheres have diameters of 6 inches and 12 inches, respectively. What is the positive difference in their volumes?

A) 36π
B) 252π
C) 288π
D) $2{,}016\pi$

HINT: For Q26, when the water is poured into a larger glass, what information about the water does not change?

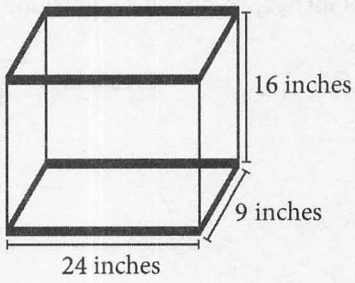
26. Alma pours water into a small cylindrical glass with a height of 6 inches and a diameter of 3 inches. The water fills the glass to the very top, so she decides to pour it into a bigger glass that is 8 inches tall and 4 inches in diameter. Assuming Alma doesn't spill any when she pours, how many inches high will the water reach in the bigger glass?

A) 1.5
B) 2.25
C) 3.375
D) 6.0



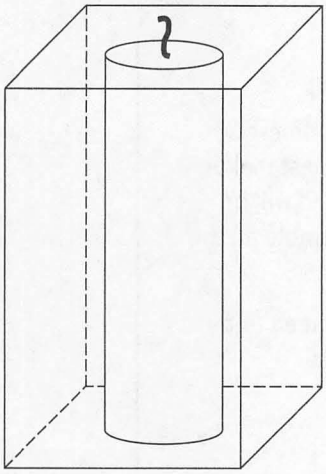
27. If the volume of the pyramid shown in the figure above can be represented by the function $V(x) = x^3 - x$, which of the following expressions represents the pyramid's height?

A) x
B) $2x$
C) $x - 1$
D) $x - 3$



28. A pet store decided to begin selling fish, so the manager purchased 50 of the fish tanks shown above to hold the fish. The staff need to fill the bottom two inches of each tank with sand, which comes only in full-size 40-pound bags. If 1 cubic inch of sand weighs 2 ounces, how many bags of sand does the pet store need to buy? (There are 16 ounces in 1 pound.)

A) 45
B) 68
C) 84
D) 125



29. In the figure above, a cylindrical candle with a diameter of 2 inches and height of 8 inches sits within a rectangular glass box. The box is the same height as the candle and the area of the base of the box is 15 square inches. If Felipe wants to fill the space between the candle and the box with wax, how many cubic inches of wax does he need?

A) $120 - 8\pi$

B) $120 - 32\pi$

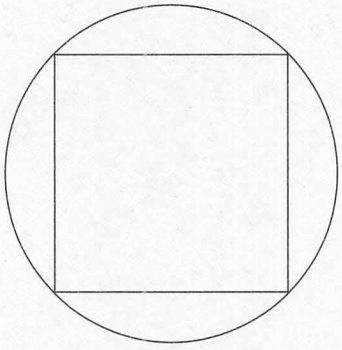
C) $225 - 8\pi$

D) $225 - 32\pi$

On Test Day

What students often find most challenging about questions involving geometry is not the need to remember or apply formulas but rather the fact that the test maker likes to "hide" the path to the solution. The starting information you need will be given, but you'll need to make deductions from that information to assemble all the facts you need to answer the question.

As you consider the question below, ask yourself what information you need to be able to solve. How can you get that information from what you're given?



30. In the figure above, a square is inscribed in a circle, and the area of the square is 400 square units. What is the area of the circle in square units?

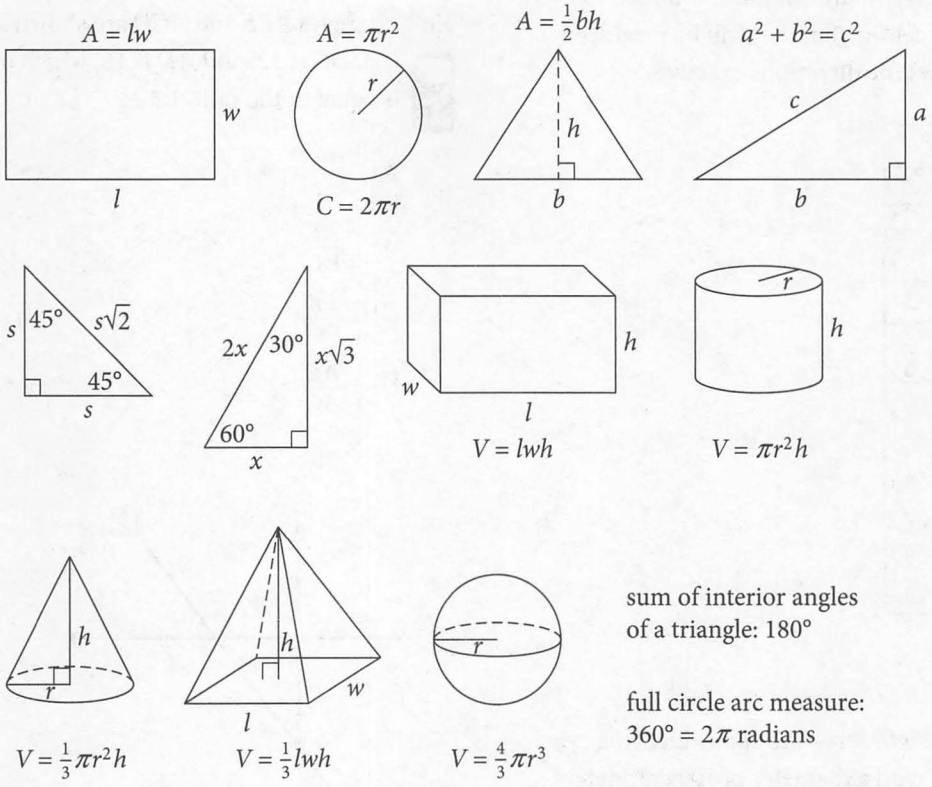
    A) 50π
    B) 100π
    C) 200π
    D) 400π

The explanation appears at the end of this chapter.

How Much Have You Learned?

Directions: For testlike practice, give yourself 22 minutes to complete this question set. Be sure to study the explanations, even for questions you got right. They can be found at the end of this chapter. Note that this question set includes foundational topics in Geometry covered in Math Fundamentals in chapter 2.

Make use of the formula sheet below as needed; you'll have these same formulas available in your test booklet when you take the real SAT.



sum of interior angles of a triangle: 180°

full circle arc measure: 360° = 2π radians

Questions 31 and 32 refer to the following stimulus.

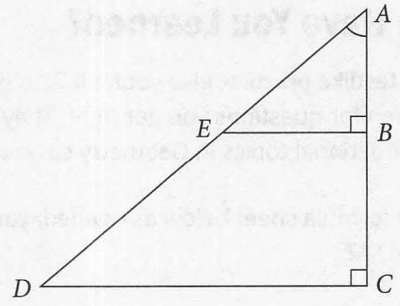
Desiree is making apple juice from concentrate. The cylindrical container of concentrate has a diameter of 7 centimeters and a height of 12 centimeters. To make the juice, the concentrate must be diluted with water so that the mix is 75 percent water and 25 percent concentrate.

31. What is the total volume of juice, rounded to the nearest cubic centimeter, that will be produced if Desiree follows the directions exactly?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

32. Desiree is going to serve the apple juice in a cylindrical pitcher with a diameter of 10 centimeters. What is the minimum height of the pitcher, rounded to the nearest centimeter, required for it to hold the apple juice that Desiree made?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9



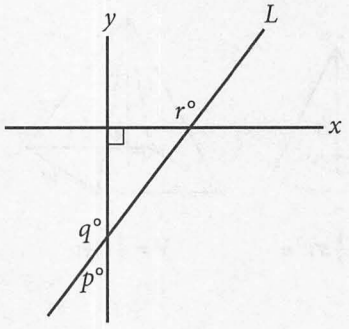
33. Triangles ABE and ACD are shown above. If AB is 3, DC is 12, and AD is 15, which of the following is equal to the ratio 1:3 ?

A) $\frac{EB}{AC}$

B) $\frac{AE}{DC}$

C) $\frac{EB}{DC}$

D) $\frac{AE}{AC}$



34. In the figure above, if $q = 140$, what is the value of $r - p$?

A) 0

B) 10

C) 90

D) 130

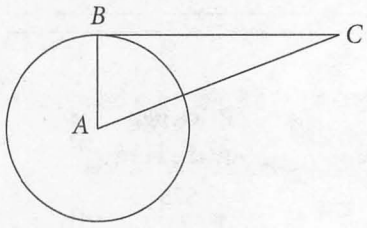
35. The area of a right triangle is 35 square inches. If the longer leg is 3 inches longer than the shorter leg, what is the length of the hypotenuse, in inches?

A) 10

B) $\frac{\sqrt{35}}{2}$

C) $7\sqrt{10}$

D) $\sqrt{149}$



36. Point A is the center of the circle and line segment BC is tangent to the circle. If the measure of $\angle A$ is $(3k + 23)°$ and the measure of $\angle C$ is $(4k - 31)°$, what is the value of k?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

37. A sphere with a diameter of 2 inches is resting on a table. If the distance from the center of the sphere to the edge of the table is 10 inches, what is the distance, in inches, between the point where the sphere contacts the table and the table edge?

A) $4\sqrt{6}$

B) $3\sqrt{11}$

C) 10

D) $\sqrt{101}$

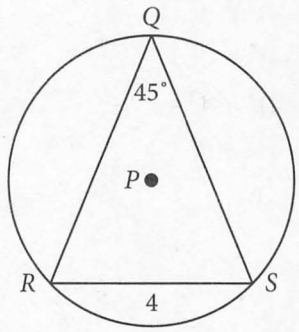
38. At Wesley's Pizzeria, the small pizza is 4 inches smaller in diameter than the large pizza, whose diameter is 16 inches. If each pizza is cut into eight equal slices, one large slice is approximately what percent larger than one small slice?

A) 44%

B) 62%

C) 78%

D) 84%



Note: Figure not drawn to scale.

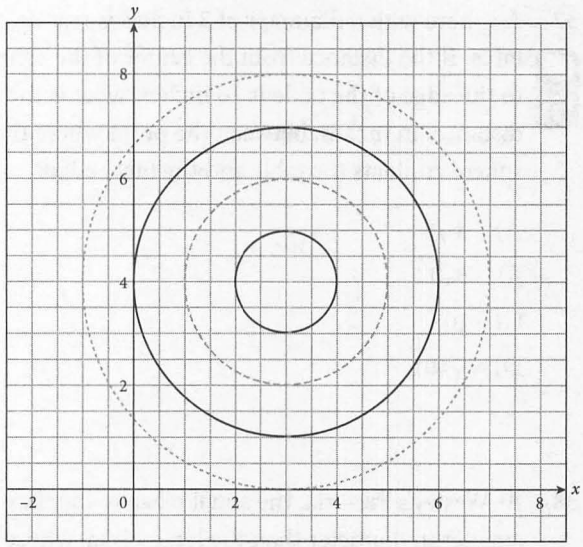
39. In circle P above, $RS = 4$ and $\angle RQS$ measures 45°. What is the circumference of circle P?

A) 4π

B) $4\pi\sqrt{2}$

C) 8π

D) $8\pi\sqrt{2}$



40. An airport instrumentation technician needs to calibrate the air traffic control radar system. An air traffic control radar system display consists of four concentric circles in a coordinate plane. The diameter of the smallest circle is two units, and the diameter of each successive circle is two units larger than that of the previous circle. The diagram above shows the radar system grid before calibration. What is the equation for the smallest dotted circle before the calibration?

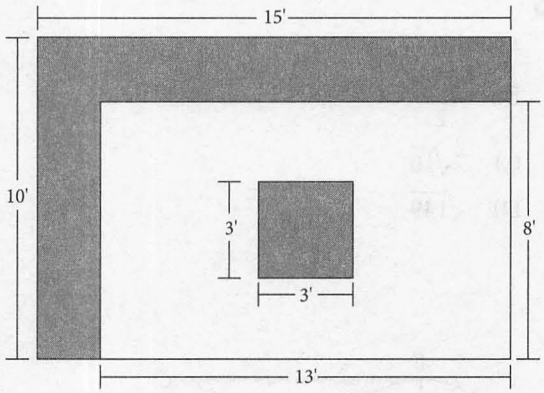
A) $(x+3)^2+(y+4)^2=4^2$

B) $(x-3)^2+(y-4)^2=2^2$

C) $(x-3)^2+(y-4)^2=4^2$

D) $(x+3)^2+(y+4)^2=8^2$

Questions 41 and 42 refer to the following information.



Material	Price per Square Foot
Quartz	$75
Stainless steel	$67
Granite	$102
Laminate	$71
Wood	$81
Concrete	$55
Soapstone	$97
Travertine	$72

41. An interior designer is trying to determine the best counter material for a kitchen remodel. The designer must cover all of the shaded counter areas in the floor plan above.

What is the area of the counter surface that must be covered, in square feet?

A) 9

B) 46

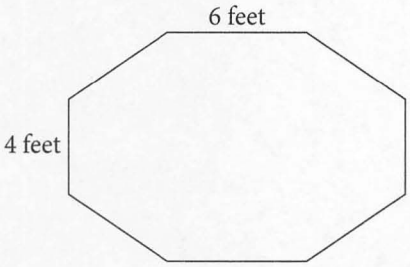
C) 55

D) 104

42. The family gives the interior designer a $4,500 budget to buy materials for the counter. A list of the available materials and the price per square foot appear in the table above. Which choice provides only options that the interior designer can present to the family that stay within the allotted budget?

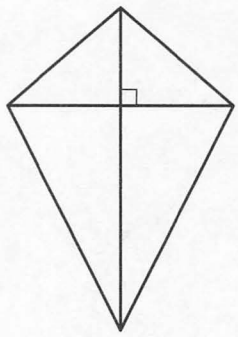
    I. Wood
    II. Laminate
    III. Soapstone
    IV. Quartz
    V. Granite

    A) I and III
    B) II, III, IV
    C) I, II, IV
    D) III and V



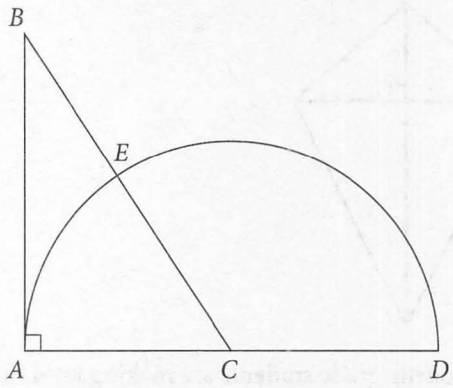
43. A carpenter creates the octagonal dining room table, shown above, by cutting an isosceles right triangle from each corner of the original piece of lumber. If all four isosceles right triangles are congruent and each has an area of $\frac{9}{2}$ square feet, what are the dimensions of the original piece of lumber, in feet?

    A) 8×8
    B) 10×12
    C) 12×14
    D) 14×18



44. Sixty fourth-grade students are making kites that are 18 inches wide and 24 inches tall. Each student uses the vertically symmetrical pattern, shown above, to cut out the kite from an 18- by 24-inch piece of material, such that the height of each kite is parallel to the longer edge of the material. After the students finish making the kites, one of the teachers collects all of the leftover kite material. If all the students successfully make their kites on the first try, how much leftover material will the teacher collect?

    A) 1.5 square feet
    B) 15 square feet
    C) 90 square feet
    D) 12,960 square feet



45. In the figure above, C is the center of the semicircle and points A, E, and D are on the semicircle. If CB equals 12 and the measure of angle ABC is 30°, what is the length of CE? (Round your answer to the nearest unit.)

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Reflect

Directions: Take a few minutes to recall what you've learned and what you've been practicing in this chapter. Consider the following questions, jot down your best answer for each one, and then compare your reflections to the expert responses on the following page. Use your level of confidence to determine what to do next.

How can you tell whether two triangles are similar?

What are the two Pythagorean triples you are most likely to see on test day?

What are the ratios of the side lengths of a 45-45-90 triangle? Of a 30-60-90 triangle?

What is the standard form for the equation of a circle?

What is the relationship of a circle's central angle to the arc and sector the angle defines?

How would you find the surface area of a right triangular prism with equilateral triangles as its bases?

Expert Responses

How can you tell whether two triangles are similar?

There are three ways to tell:

- *Two of their three angles are congruent (angle-angle).*
- *Two of their three sides are in the same proportion and the intervening angle is congruent (side-angle-side).*
- *Their three sides are in the same proportion (side-side-side).*

What are the two Pythagorean triples you are most likely to see on test day?

The two most common Pythagorean triples on the SAT are 3:4:5 and 5:12:13. You may also see multiples of these, e.g., 6:8:10 or 10:24:26.

What are the ratios of the side lengths of a 45-45-90 triangle? Of a 30-60-90 triangle?

The side lengths of a 45-45-90 triangle are always in the ratio of $x:x:x\sqrt{2}$. *The side lengths of a 30-60-90 triangle are always in the ratio of* $x:x\sqrt{3}:2x$. *Remember that the shortest side of any triangle is across from the smallest angle and the longest side is across from the greatest angle.*

What is the standard form for the equation of a circle?

The equation of a circle in the coordinate plane is $(x - h)^2 + (y - k)^2 = r^2$, *where* r *is the radius of the circle and* (h, k) *is the ordered pair representing its center.*

What is the relationship of a circle's central angle to the arc and sector the angle defines?

The central angle, the arc, and the sector are all in proportion to the full circle:

$$\frac{\text{central angle}}{360^\circ} = \frac{\text{arc length}}{\text{circumference}} = \frac{\text{sector area}}{\text{circle area}}$$

How would you find the surface area of a right triangular prism with equilateral triangles as its bases?

Calculate the area of one of the equilateral triangles and multiply by 2. Calculate the area of one of the rectangular faces and multiply by 3. Then add the results.

Next Steps

If you answered most questions correctly in the "How Much Have You Learned?" section, and if your responses to the Reflect questions were similar to those of the SAT expert, then consider Geometry an area of strength and move on to the next chapter. Come back to this topic periodically to prevent yourself from getting rusty.

If you don't yet feel confident, review those parts of this chapter that you have not yet mastered. In particular, review the lessons on similar triangles, the Pythagorean theorem, and special right triangles, as these are high-yield topics on the SAT. Then try the questions you missed again. As always, be sure to review the explanations closely. Finally, **go online** (www.kaptest.com/moreonline) for additional practice on the highest yield topics in this chapter.

Answers and Explanations

1. 426

Difficulty: Medium

Strategic Advice: The figure contains a pair of similar triangles. Use the fact that their sides are in proportion to find the required length.

Getting to the Answer: The question asks for the length of *DE*. Note that $DC + DE = EC$, which means $DE = EC - DC$. First, find the length of *EC* by setting up a proportion where *BD* is related to *AC* in the same way that *DC* is related to the unknown *EC*. *AC* is 324 and *DC* is 50% longer than segment *BD*, or $1.5 \times 40 = 60$ feet:

$$\frac{BD}{AE} = \frac{DC}{EC}$$
$$\frac{40}{324} = \frac{60}{EC}$$
$$\frac{10}{81} = \frac{60}{EC}$$
$$10x = 4{,}860$$
$$EC = 486$$

Subtract the length of *DC*, 60, from the length of *EC*, 486, to obtain 426, the length of *DE*. Grid in **426**.

2. B

Difficulty: Medium

Getting to the Answer: Given the area of $\triangle ABC$ and the length of the base *AB*, you can find *BC*, its height:

$$150 = \frac{1}{2}(20)(BC)$$
$$150 = 10(BC)$$
$$15 = BC$$

Because lengths $BC = 15$ and $AB = 20$, $\triangle ABC$ is a 3:4:5 triangle with dimensions scaled up by a factor of 5. The hypotenuse, *AC*, must be $5 \times 5 = 25$. $\triangle ABC$ and $\triangle AGH$ are similar triangles because they share an angle at vertex *A* and they each have a right angle. Therefore, their corresponding sides must be proportional. The question says that the hypotenuse of $\triangle AGH$, *AH*, is 20, so use this information to create a proportion:

$$\frac{AH}{AC} = \frac{HG}{CB}$$
$$\frac{20}{25} = \frac{HG}{15}$$
$$300 = 25HG$$
$$HG = 12$$

Choice **(B)** is the correct answer.

3. B

Difficulty: Medium

Getting to the Answer: The question says that segment *HI* is the bisector of segments *LO* and *OW*. This tells you two things: 1) *HI* divides both *LO* and *OW* exactly in half and 2) *HI* is parallel to *LW*.

Because *HI* is parallel to *LW*, angles *L* and *H* must be congruent (they are corresponding angles) and angles *W* and *I* must be congruent (they are also corresponding angles). Angle *O* is shared by both triangles. The triangles, therefore, are similar. Side lengths of similar triangles are in proportion to one another. Because *I* is the midpoint of *OW*, *OI* is half as long as *OW*. (The same is true for the other side: *OH* is half as long as *OL*.) So the sides are in the ratio 1:2. The question gives the side lengths of *LW* and *HI*. Use this ratio and these side lengths to set up a proportion and solve for *x*:

$$\frac{1}{2} = \frac{HI}{LW}$$
$$\frac{1}{2} = \frac{4x - 1}{30}$$
$$30 = 2(4x - 1)$$
$$30 = 8x - 2$$
$$32 = 8x$$
$$4 = x$$

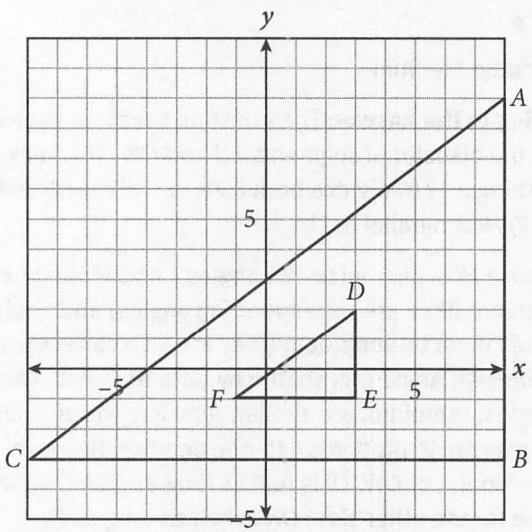
The correct answer is **(B)**.

4. 9

Difficulty: Hard

Getting to the Answer: Corresponding sides of similar triangles are proportional. Draw a quick sketch to find as many side lengths as you can, find the ratio of the sides between the two triangles, and use that ratio to find the missing vertex.

Plot all the points given in the question, labeling them as you go so you don't get confused (especially because you won't have graph paper). You know that *D* and *A* are in the same quadrant, which means the triangles are both oriented the same way, so make your sketch accordingly:



Once you have plotted triangle *DEF* and the base of triangle *ABC*, you can determine that the ratio of the triangles is 1:4 (the base of *DEF* has a length of 4 and the base of *ABC* has a length of 16). To determine where you should put *A*, find the length of side *DE* and then multiply by 4. The length of the vertical side of triangle *ABC* is $3 \times 4 = 12$. Because one vertex is at $(8, -3)$, vertex *A* must be 12 vertical units above that point, or $(8, 9)$. The *y*-coordinate of *A* is **9**.

5. B

Difficulty: Hard

Getting to the Answer: The figure gives two triangles that look as though they have a lot in common. Analyze them, looking for similarities so you can set up a proportion. Angle *C* is shared by both triangles. And, because *BD* is parallel to *AE*, angles *CAE* and *CBD* are congruent—they are corresponding angles. Two pairs of congruent angles means that triangles *ACE* and *BCD* are similar by angle-angle. (You could have analyzed angles *CDB* and *CEA*, but you need only two pairs of congruent angles to conclude that two triangles are similar.)

Set up a proportion using the triangles' side lengths. You'll need to translate from English into math as you go: $AB = 5$ and *BC* is three times that, or 15. This means $AC = 5 + 15 = 20$. *CD* is 2 more than half *AC*, $\frac{20}{2} + 2 = 10 + 2 = 12$. Use the three known side lengths to create a proportion and solve for *EC*:

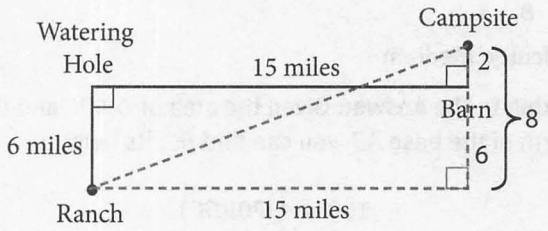
$$\frac{BC}{AC} = \frac{DC}{EC}$$
$$\frac{15}{20} = \frac{12}{EC}$$
$$15EC = 240$$
$$EC = 16$$

The question asks for the length of segment *DE*, which is $EC - CD$, or $16 - 12 = 4$. **(B)** is correct.

6. A

Difficulty: Medium

Getting to the Answer: Start by connecting the ranch to the campsite. Then draw in a horizontal line and a vertical line to form a right triangle.



The length of one leg of the triangle is 15 miles, the distance from the watering hole to the barn. The length of the other leg is $6 + 2 = 8$ miles, the distance from the ranch to the watering hole plus the distance from the barn to the campsite. The two legs of the right triangle are 8 and 15. You might recognize the Pythagorean triple 8:15:17, but if you don't, you can always rely on the Pythagorean theorem:

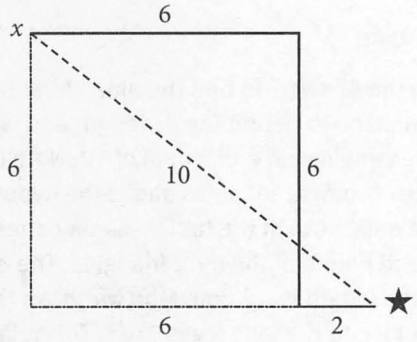
$$8^2 + 15^2 = c^2$$
$$64 + 225 = c^2$$
$$289 = c^2$$
$$\sqrt{289} = \sqrt{c^2}$$
$$17 = c$$

The actual trail is $6 + 15 + 2 = 23$ miles long. The direct route is 17 miles, so the direct route is $23 - 17 = 6$ miles shorter, so **(A)** is the correct answer.

7. 4

Difficulty: Medium

Getting to the Answer: Start by drawing Ted's car and bike routes to his friend's house and labeling your diagram with the distances you know. To find the distance of Ted's bike route, create a right triangle:

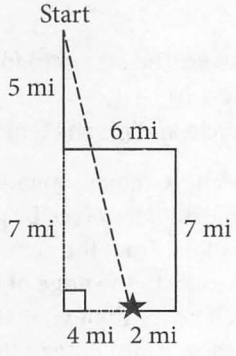


You can use the Pythagorean theorem to find the hypotenuse (bike route), but you'll save time if you recognize that you have a Pythagorean triple (6:8:10, a common multiple of 3:4:5). Thus, Ted's bike route is $6 + 6 + 2 = 14$ miles, and his car route is 10 miles; the difference between the two is **4**.

8. A

Difficulty: Medium

Getting to the Answer: The question asks for the total distance they traveled. Aundria traveled $5 + 6 + 7 + 2 = 20$ miles. To find the distance Annette traveled, draw and label a diagram of Aundria and Annette's routes and look for a place to sketch in a right triangle so that Annette's direct route is the hypotenuse:



Use the Pythagorean theorem to calculate the distance Annette will travel:

$$c^2 = (5+7)^2 + 4^2$$
$$c^2 = 144 + 16$$
$$c^2 = 160$$
$$c = \sqrt{160}$$

Note that the answers are in decimal form, so convert the root value to a decimal: about 12.65 miles. Add this to Aundria's travel distance to find the total distance: $20 + 12.65 = 32.65$ miles. Choice **(A)** is correct.

9. 4

Difficulty: Medium

Getting to the Answer: Use the Pythagorean equation to solve for x:

$$(3x+1)^2 = (3x)^2 + (x+1)^2$$
$$9x^2 + 6x + 1 = 9x^2 + x^2 + 2x + 1$$
$$6x = x^2 + 2x$$
$$4x = x^2$$
$$4 = x$$

(Note that x represents a distance and cannot be negative, so -4 is not a viable solution. Moreover, you cannot grid in a negative number.)

Grid in **4**.

10. D

Difficulty: Hard

Getting to the Answer: To calculate the area of the semicircle, you need to find the length of the radius, which is AE. Since $AE = EC$, AE is half of AC. Start by solving for the length of AC, which is the hypotenuse of right triangle ABC. Note that the sides of the triangle are a Pythagorean triple: 5, 12, 13. If you did not recognize the triple, you could have solved for AC using the Pythagorean theorem: $AC = \sqrt{5^2 + 12^2} = 13$. Thus, $AC = 13$ and $AE = \frac{13}{2}$. The area of the semicircle is therefore $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{13}{2}\right)^2 = \frac{169}{8}$. **(D)** is correct.

11. C

Difficulty: Medium

Getting to the Answer: Triangle *DBC* is an isosceles right triangle, which means that it is a 45-45-90 right triangle with side length ratios of $x{:}x{:}x\sqrt{2}$. Because the two legs are each 4 units long, the hypotenuse *DB* is $4\sqrt{2}$. *DB* also acts as one leg of right triangle *DAB*. Since this leg is $4\sqrt{2}$ and the hypotenuse is $5\sqrt{2}$, triangle *DAB* is a 3:4:5 right triangle, and the length of *AD* is $3\sqrt{2}$. The legs of a right triangle are also its base and height, so plug the lengths of the legs into the formula for the area of a triangle, $\frac{1}{2}bh$, to get $\frac{1}{2}\left(4\sqrt{2}\right)\left(3\sqrt{2}\right) = 6 \times 2 = 12$. The correct answer is **(C)**.

12. B

Difficulty: Easy

Getting to the Answer: The height of the amusement ride at point *M* is perpendicular to the ground, and the roller coaster's angle of ascent is 30°, which creates a 30-60-90 triangle. Use the ratio of the sides $x{:}x\sqrt{3}{:}2x$ to find the height at point *M*. Because the distance from point *O* to point *M* is $200\sqrt{3}$ and corresponds to the hypotenuse of the triangle, then $200\sqrt{3} = 2x$. Solving for *x* by dividing both sides by 2 gives the length of the side opposite the 30° angle, which corresponds to the height of the amusement ride: $x = 100\sqrt{3}$ feet. **(B)** is correct.

13. C

Difficulty: Medium

Getting to the Answer: The top face of the corner shelf is a 45-45-90 triangle. Define the length of the shorter side, say *s*, and use the ratio of the sides $x{:}x{:}x\sqrt{2}$ to solve for it:

$$\frac{1}{\sqrt{2}} = \frac{s}{10}$$
$$s\sqrt{2} = 10$$
$$s = \frac{10}{\sqrt{2}}$$

Next, determine the top face surface area of one corner shelf using $\frac{1}{2}bh$. Note that in a right triangle, the sides that form the 90° angle are the base and height. Thus, the surface area of one corner shelf is $\frac{1}{2}\left(\frac{10}{\sqrt{2}}\right)\left(\frac{10}{\sqrt{2}}\right) = \frac{100}{4} = 25$ square inches. The question asks for the total surface area of three corner shelves, so $3 \times 25 = 75$ square inches. Choice **(C)** is correct.

Alternatively, you could find the altitude perpendicular to the longest side. Note the triangle can be split into two smaller 45-45-90 triangles. Thus, the altitude is the leg of the smaller 45-45-90 triangle, which is half of 10. The top face surface area of one corner shelf is therefore $\frac{1}{2}(10)(5) = 25$ square inches, and the total surface area for the three shelves is $3 \times 25 = 75$ square inches.

14. D

Difficulty: Hard

Getting to the Answer: To find the area of the tablet screen, you need to determine its length and width. Sketch a rectangle with a diagonal of 12. Note the diagonal creates two right triangles and is the hypotenuse. The length and width of the tablet screen correspond to the base and height of the right triangles. The question says that the length is $\sqrt{3}$ times the width, so the ratio of the short leg to the long leg is $1{:}\sqrt{3}$. This relationship is part of the ratio of the side lengths of a 30-60-90 triangle: $x{:}x\sqrt{3}{:}2x$. The hypotenuse is 12, which corresponds to $2x$, so $x = 6$. Therefore, the width is 6, and the length is $\sqrt{3} \times 6 = 6\sqrt{3}$. Use $l \times w$ to calculate the area: $6\sqrt{3} \times 6 = 36\sqrt{3}$ square inches. The correct answer is **(D)**.

15. 30

Difficulty: Medium

Getting to the Answer: Sketch a right triangle with a height of 2 and hypotenuse of 4 to represent the ramp to the stage. The ratio of the short side to the hypotenuse is 2:4, or 1:2. Notice that this matches the ratio of the sides for a 30-60-90 triangle: $x{:}x\sqrt{3}{:}2x$. The angle of incline is opposite the short side, which is 30°. Grid in **30**.

16. D

Difficulty: Easy

Getting to the Answer: The standard form of the equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the center of the circle and *r* is the length of the radius.

The answer choices have many similarities, which will make them easy to eliminate piece by piece. Use the graph to find the radius. From the center, you can count horizontally or vertically to the edge of the circle to find that its radius is 6. If $r = 6$, then $r^2 = 36$. Eliminate (A) and (B). Now find the *x*-coordinate of the center of the circle, -1. This means the $(x-h)^2$ part of the equation

is $(x-(-1))^2=(x+1)^2$. Eliminate (C). Only **(D)** is left and is correct. Note that you do not even need to find the $(y-k)^2$ part of the equation, but for the record: because the *y*-coordinate of the center of the circle is 2, then $(y-k)^2$ becomes $(y-2)^2$, and the full equation is $(x+1)^2+(y-2)^2=36$. **(D)** is indeed correct.

17. 7

Difficulty: Hard

Getting to the Answer: Rewrite the given equation for the circle in standard form to find the coordinates of the center. Start by completing the square for each of the variables. Take half of the coefficient and then square it. For the *x*-coefficient, half of 6 is 3, and 3 squared is 9. For the *y*-coefficient, half of 8 is 4, and 4 squared is 16. Add 9 and 16 to both sides and rewrite as two separate squares of binomials:

$$(x-h)^2+(y-k)^2=r^2$$
$$x^2+6x+y^2-8y=171$$
$$x^2+6x+9+y^2-8y+16=171+9+16$$
$$(x+3)^2+(y-4)^2=196$$

The coordinates of the center of the circle are $(-3, 4)$. The positive difference is $4-(-3)=$ **7**. Grid in **7**. If you came up with a negative number, remember that the answer to a Grid-in question cannot be negative.

18. B

Difficulty: Hard

Getting to the Answer: The question asks for the diameter, which is twice the radius. When the equation of a circle is in the form $(x-h)^2+(y-k)^2=r^2$, the *r* represents the length of the radius. The question gives the equation in general form, so you need to complete the square to put the equation into standard form.

You already have an x^2 and a y^2 in the given equation and the coefficients of *x* and *y* are even, so completing the square is fairly straightforward—there are just a lot of steps. Start by grouping the *x*'s and *y*'s together. Then, take the coefficient of the *x* term and divide it by 2, square it, and add it to the two terms with *x* variables. Do the same with the *y* term. Remember to add these amounts to the other side of the equation as well. Then factor the perfect squares and simplify:

$$x^2+y^2+8x-20y=28$$
$$x^2+8x+y^2-20y=28$$
$$\left(x^2+8x+16\right)+\left(y^2-20y+100\right)=28+16+100$$
$$(x+4)^2+(y-10)^2=144$$

The equation tells you that r^2 is 144, which means that the radius is 12 and the diameter is twice that, or 24, which is **(B)**.

19. D

Difficulty: Medium

Getting to the Answer: The given equation defines a circle with its center at $(4, -2)$ and a radius of 10. Any point on the circumference of the circle must satisfy that equation. Eliminate (C) because it is the center of the circle. Plug the other choices into the equation to see if they satisfy the given equation:

(A) $(-3-4)^2+(5+2)^2=(-7)^2+(7)^2=49+49=98$. Since $98<100$, this is inside the circle. Eliminate (A).

(B) $(0-4)^2+(9+2)^2=(-4)^2+(11)^2$. Since the second term alone is greater than 100, this is outside the circle. Eliminate (B).

Only **(D)** is left and is correct. For the record:

(D) $(4-4)^2+(8+2)^2=(0)^2+(10)^2=100$. This point satisfies the equation and is therefore on the circumference. **(D)** is indeed correct.

20. A

Difficulty: Medium

Getting to the Answer: To find a central angle based on a known arc length, use the relationship $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360°}$. The unknown in the relationship is the central angle, so call it a. Before you can fill in the rest of the equation, you need to find the circumference of the circle: $C = 2\pi r = 2\pi(120) = 240\pi$. Now, you're ready to solve for a:

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360°}$$
$$\frac{200}{240\pi} = \frac{a}{360°}$$
$$\frac{200(360)}{240\pi} = a$$
$$95.5° \approx a$$

(A) is correct. Be careful when you enter this expression into your calculator—you need to put 240π in parentheses so that the calculator doesn't divide by 240 and then multiply by π.

21. C

Difficulty: Medium

Getting to the Answer: Because the ratio of the shaded area to the non-shaded area is 4:5, the ratio of the shaded area to the entire circle is $4:(4+5) = 4:9$. This ratio is the same as the ratio of the interior angle of the shaded sector to 360°, or x:360. Set up a proportion using these ratios:

$$\frac{4}{9} = \frac{x}{360}$$
$$360(4) = 9x$$
$$1{,}440 = 9x$$
$$160 = x$$

Choice **(C)** is correct.

22. B

Difficulty: Medium

Getting to the Answer: The measure of an arc is directly related to the degree measure of its central angle. The measure of an inscribed angle is half of that of the central angle. Because the measure of arc *KML* is 150°, the degree measure of the inscribed angle is half of that. The inscribed angle for arc *KML* can be written as $3x + 2x$. Set up an equation to solve for x:

$$3x + 2x = \frac{1}{2}(150)$$
$$5x = 75$$
$$x = 15$$

Choice **(B)** is correct.

23. C

Difficulty: Hard

Getting to the Answer: Note that the figure is composed of two triangles inscribed in a circle. If you have enough information about the triangles, you can find the length of *WX*. Arcs *XY* and *YZ* are each 60°. Because an inscribed angle is half of its corresponding arc, the inscribed angles *XWY* and *ZWY* are each 30°. Similarly, the sum of inscribed angles *XYW* and *ZYW* is half of arc *XWZ*: $\frac{1}{2}(360 - (60 + 60)) = 120$. Thus, angles *XYW* and *XWY* are each 60°. Therefore, both triangles are 30-60-90 special right triangles, so use the ratio of the sides $x{:}x\sqrt{3}{:}2x$ to find the length of *WX*. Given $YZ = 3$, *XY* also equals 3 and $\frac{3}{WX} = \frac{1}{\sqrt{3}}$, so $WX = 3\sqrt{3}$. Choice **(C)** is correct.

24. D

Difficulty: Medium

Getting to the Answer: Since *BC* is the diameter, the measure of arc *BAC* is 180°. Therefore, the measure of arc *BA* is the measure of arc *BAC* minus the measure of arc *AC*: $180° - 30° = 150°$. Use $180° = \pi$ to convert to radians: $150° \times \frac{\pi}{180°} = \frac{5\pi}{6}$. Thus, **(D)** is correct.

25. B

Difficulty: Medium

Getting to the Answer: Begin by finding the volume of each sphere using the volume formula for a sphere, remembering to halve the diameters first:

$$V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$$
$$V_2 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216) = 288\pi$$

The positive difference is $288\pi - 36\pi = 252\pi$, which is **(B)**.

26. C

Difficulty: Medium

Getting to the Answer: After the water is poured into the larger glass, the volume of the water in the glass will be the same as the volume when it was in the smaller glass. Find the volume of the water in the smaller glass, whose height is 6 inches and radius is 3 inches. Then, substitute this volume into a second equation where the height is unknown and the radius is 4 inches (the radius of the larger glass) and solve for h. The volume of a cylinder equals the area of its base times its height, or $V = \pi r^2 h$:

$$V = \pi r^2 h$$
$$V = \pi(1.5)^2(6)$$
$$V = \pi(2.25)(6)$$
$$V = 13.5\pi$$

$$13.5\pi = \pi(2)^2 h$$
$$13.5\pi = 4\pi h$$
$$3.375 = h$$

The water will reach 3.375 inches high in the bigger glass. **(C)** is correct.

27. C

Difficulty: Medium

Getting to the Answer: The formula for finding the volume of a pyramid with a rectangular base is $V = \frac{1}{3}lwh$. Start by substituting what you know into the formula. The volume is represented by $x^3 - x$, the length is $x + 1$, and the width is $3x$:

$$V = \frac{1}{3}lwh$$
$$x^3 - x = \frac{1}{3}(x+1)(3x)h$$
$$x\left(x^2 - 1\right) = (x+1)xh$$

Notice that if you divide both sides of the equation by x, you'll be left with $x^2 - 1$ on the left side and $(x + 1)$ times h on the right side. Note that this is a classic quadratic: the factors of $x^2 - 1$ are $x + 1$ and $x - 1$. So:

$$(x+1)(x-1) = (x+1)h$$
$$x - 1 = h$$

This means the height of the pyramid must be represented by $x - 1$. Therefore, **(C)** is correct.

28. B

Difficulty: Hard

Getting to the Answer: When a question involves many steps, as this one does, plan out the order of your calculations and conversions. In this case, you can go from volume of sand in 1 tank to volume of sand in 50 tanks, to weight of sand in ounces, to weight of sand in pounds, to the number of bags of sand.

The volume of sand in one tank (only 2 inches of the height) will be $V = 24 \times 9 \times 2 = 432$ cubic inches, which means the volume of sand in all 50 tanks will be $50 \times 432 = 21{,}600$ cubic inches. Each cubic inch of sand weighs 2 ounces, so the weight of all the sand will be $2 \times 21{,}600 = 43{,}200$ ounces. There are 16 ounces in 1 pound, so the weight of the sand in pounds is $43{,}000 \text{ ounces} \times \frac{1 \text{ pound}}{16 \text{ ounces}} = 2{,}700$ pounds. Finally, each bag contains 40 pounds of sand, so the pet store needs to buy $2{,}700 \text{ pounds} \times \frac{1 \text{ bag}}{40 \text{ pounds}} = 67.5$ bags. Because the store cannot buy one-half of one 40-pound bag of sand, it will need to buy 68 bags of sand. **(B)** is the correct answer.

29. A

Difficulty: Medium

Getting to the Answer: Determine the volumes of the rectangular box and of the cylindrical candle. Then calculate the difference between the two to find the volume of space between them. The volume of the box is the area of the base times its height: $15 \times 8 = 120$ cubic inches. Eliminate (C) and (D). Next, use $\pi r^2 h$ to calculate the volume of the cylinder. The diameter is 2 inches, so the radius is 1 inch: $\pi(1)^2(8) = 8\pi$. The difference of the volumes gives you **(A)**. Felipe needs $120 - 8\pi$ cubic inches of wax.

30. C

Difficulty: Hard

Category: Special Right Triangles

Strategic Advice: The diagonal of the square divides it into two 45-45-90 triangles. Use this information to find the diagonal of the square, which is also the diameter of the circle. From there, you can calculate the area of the circle.

Getting to the Answer: The area of the square is 400, so each side of the square is $\sqrt{400} = 20$. The sides of a 45-45-90 triangle are in the ratio $x{:}x{:}x\sqrt{2}$, so the diagonal of the square is $20\sqrt{2}$ and the radius of the circle is half of that, or $10\sqrt{2}$. The area of the circle is then $\pi\left(10\sqrt{2}\right)^2 = \pi(10)^2\left(\sqrt{2}\right)^2 = \pi(100)(2) = 200\pi$.

Choice **(C)** is correct.

31. 1847

Difficulty: Medium

Category: Three-Dimensional Figures

Getting to the Answer: The question asks for the total volume of juice, which is the volume of the cylinder that contains the concentrate plus the amount of water that is added. The height of the can of concentrate is 12 centimeters, and its diameter is 7 centimeters, making the radius 3.5 centimeters. Plug this information into the formula for the volume of a cylinder, $V = \pi r^2 h$: $V_{conc} = \pi \times 3.5^2 \times 12 = 147\pi \text{ cm}^3$.

The ratio of concentrate to total juice is $V_{conc}{:}(V_{conc} + V_{water})$ or $25{:}(25 + 75)$, which simplifies to 1:4. Set up the following ratio to calculate the total volume:

$$\frac{147\pi}{V_{Total}} = \frac{1}{4}$$

Cross-multiply to find the total volume of juice:

$$V_{Total} = (147\pi)(4)$$
$$V_{Total} = 588\pi \approx 1{,}847.2$$

The value 1,847.2 rounds to 1,847. Grid in **1847**.

32. 24

Difficulty: Medium

Category: Three-Dimensional Figures

Getting to the Answer: The diameter of the serving pitcher is given as 10 centimeters, so the radius is 5 centimeters. Use the radius and the total volume of apple juice you found in the last question to find the minimum height of the serving pitcher:

$$V_{Total} = 588\pi = \pi\left(5^2\right)(\text{height})$$
$$588\pi = \pi(25)(\text{height})$$
$$588 = 25(\text{height})$$
$$23.52 = \text{height}$$

The height needs to be rounded up to **24**, which is the correct answer.

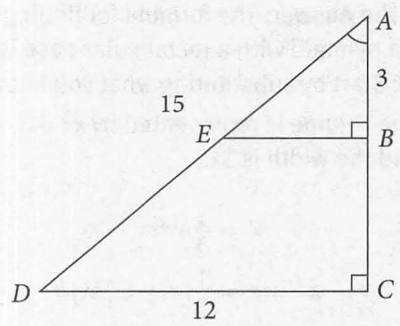
33. C

Difficulty: Medium

Category: Similar Triangles; Special Right Triangles

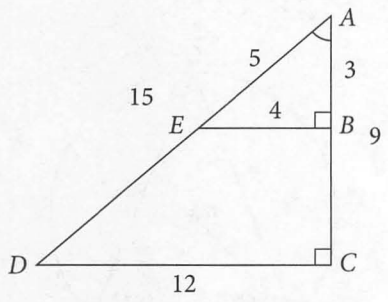
Strategic Advice: For questions with right triangles, look for Pythagorean triples and special right triangles.

Getting to the Answer: Begin by transferring all of the information in the question stem to the diagram:



Triangle *ACD* is a right triangle with side length 12 and a hypotenuse of 15, which means its other side length, *AC*, is 9. These side lengths are the 3:4:5 triple multiplied by 3. Because they share angle *A* and both have a right angle, the two triangles are similar by way of angle-angle. Therefore, triangle *ABE* is also a 3:4:5 triangle. Since triangle *ACD* is three times the size of triangle *ABE*, you can easily create a ratio of 1:3. Look for a side of *ABE* in the numerator and the corresponding side of *ACD* in the denominator. Only **(C)** correctly matches up the corresponding sides of *ABE* and *ACD*.

Another option to solve this question is to plug in the values for all of the sides based on the Pythagorean triples and plug those lengths into the answer choices. The diagram below shows the side lengths of the two triangles:



Plugging the lengths into the choices results in:

(A): $\frac{EB}{AC} = \frac{4}{9}$

(B): $\frac{AE}{DC} = \frac{5}{12}$

(C): $\frac{EB}{DC} = \frac{4}{12} = \frac{1}{3}$

There is no need to check (D) because there will be only one set of lengths that work. **(C)** is indeed correct.

34. C

Difficulty: Easy

Category: Lines and Angles

Getting to the Answer: Line L forms a right triangle whose right angle is created by the intersection of the x- and y-axes. The figure shows that q and p are complementary, as they form a straight line, the y-axis, which means $q + p = 180$. The question says that $q = 140$, so $p = 180 - 140 = 40$. One of the interior angles of the triangle, the one created by the y-axis and line L, is vertical to p, so that angle also is 40. Now, find the last angle measure inside the triangle by subtracting: $180 - 90 - 40 = 50$. This angle is supplementary to r, so $r = 180 - 50 = 130$. This means that $r - p = 130 - 40 = 90$. **(C)** is the correct.

35. D

Difficulty: Hard

Category: Pythagorean Theorem

Getting to the Answer: In a right triangle, one leg is the base and the other is the height. Because the question states that one leg is 3 inches longer than the other, use x and $x + 3$ to represent the lengths of these two legs. Use the formula for the area of a triangle to solve for x, the shorter leg:

$$35 = \frac{1}{2}(x)(x+3)$$

$$2(35) = \cancel{2}\left(\frac{1}{\cancel{2}}(x)(x+3)\right)$$

$$70 = (x)(x+3)$$

$$70 = x^2 + 3x$$

Next, subtract 70 to make the equation equal 0. Use reverse-FOIL to find that the factors are $(x + 10)$ and $(x - 7)$, which means $x = -10$ and $x = 7$. Lengths cannot be negative, so the shorter leg has a length of 7, and the longer leg has a length of $7 + 3 = 10$. Now, use the Pythagorean theorem to find the length of the hypotenuse:

$$a^2 + b^2 = c^2$$

$$7^2 + 10^2 = c^2$$

$$49 + 100 = c^2$$

$$149 = c^2$$

$$\sqrt{149} = \sqrt{c^2}$$

$$\sqrt{149} = c$$

Choice **(D)** is correct.

36. 14

Difficulty: Medium

Category: Pythagorean Theorem

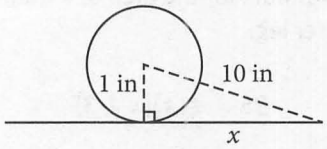
Getting to the Answer: Since a line tangent to a circle is perpendicular to a radius drawn to meet the tangent line, $\triangle ABC$ must be a right triangle. Therefore, the sum of the measures of $\angle A$ and $\angle C$ is 90°, and you can write the equation $(3k + 23) + (4k - 31) = 90$. This simplifies to $7k - 8 = 90$ and then to $7k = 98$. Divide both sides by 7 to see that $k = 14$. Grid in **14**.

37. B

Difficulty: Medium

Category: Pythagorean Theorem

Getting to the Answer: Draw a diagram to visualize the situation. A cross section of the sphere resting on the table is shown. Note that the sphere is tangent to the table and that a right triangle is present:



Plug the values from your diagram into the Pythagorean equation: $1^2 + x^2 = 10^2$. Solving for x results in $x = \sqrt{99} = \sqrt{9 \times 11} = 3\sqrt{11}$, so **(B)** is correct.

38. C

Difficulty: Medium

Category: Arc Length and Sectors

Strategic Advice: You can save time solving this question if you recognize that information about 8 slices is unnecessary. The whole large pizza will be the same percent larger than a whole small pizza as a large slice will be compared to a small slice. In other words, if a large slice is 25% larger than a small slice, a large pizza is also 25% larger than a small pizza. That means you need to calculate only the areas of each pizza and plug them into the percent change formula.

Getting to the Answer: The radius of the large pizza is one-half of its diameter of 16 inches, or 8 inches. Use the radius to find the area of the large pizza: $\pi r^2 = \pi(8)^2 = 64\pi$ square inches. The question states that the small pizza is 4 inches smaller in diameter than the large pizza, or $16 - 4 = 12$ inches. The radius of the small pizza is one-half of its diameter of 12 inches, or 6 inches, and the small pizza has an area of $\pi(6)^2 = 36\pi$ square inches.

The question asks by what percent is the large pizza larger than the small pizza, which means that the percent change is in terms of the small pizza. Plug the values in:

$$\frac{64\pi - 36\pi}{36\pi} = \frac{28\pi}{36\pi} = \frac{7}{9} \approx 78\%$$

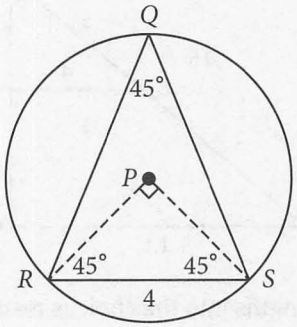
(C) is correct.

39. B

Difficulty: Medium

Category: Special Right Triangles

Getting to the Answer: Start by drawing in *PR* and *PS* as shown here:



Because the angle formed by *PR* and *PS* defines the same arc as the angle formed by segments *QR* and *QS*, $\angle RPS$ must be twice $\angle RQS$, which is $45° \times 2 = 90°$. In addition to being legs of $\triangle PRS$, *PR* and *PS* are radii, so they are congruent, which makes $\triangle PRS$ a 45-45-90 triangle. Therefore, the radius of circle *P* is $\frac{4}{\sqrt{2}} = 2\sqrt{2}$, and the circumference is $2\pi \times 2\sqrt{2} = 4\pi\sqrt{2}$, so **(B)** is correct.

40. B

Difficulty: Medium

Category: Circles

Getting to the Answer: To find the equation of the smallest dotted circle on the diagram, you need the standard form of the equation of a circle, $(x - h)^2 + (y - k)^2 = r^2$, the coordinates of the center of the circle, and its radius. The center of the circles before calibration is $(3, 4)$. Eliminate (A) and (D). The diameter of the smallest dotted circle is 4 units, which makes its radius 2 units.

Plugging that information into the circle equation results in $(x - 3)^2 + (y - 4)^2 = 2^2$. **(B)** is correct.

41. C

Difficulty: Medium

Category: Quadrilaterals

Getting to the Answer: To find the area of the counter surface that needs to be covered, add the area of the L-shaped portion of the counter and the square portion in the middle. Determine the area of the L-shaped portion of the counter by subtracting the area of the smaller 8 foot by 13 foot rectangle from the larger 10 foot by 15 foot rectangle:

$$\text{Area of L-shaped portion} = (10 \times 5) - (8 \times 13) = 150 - 104 = 46 \text{ ft}^2$$

Finally, to find the total area add the area of the 3 foot by 3 foot square in the center, $3 \times 3 = 9$ ft^2, to the 46 ft^2: $9 + 46 = 55$ ft^2. **(C)** is correct.

42. C

Difficulty: Medium

Category: Quadrilaterals

Getting to the Answer: From the answer to the previous question, you already know the area that needs to be covered, 55 square feet. Divide the budget by the area to find the maximum amount that can be spent per square foot:

$$\frac{\$4{,}500}{55 \text{ square feet}} \approx \$82 \text{ per square foot}$$

Instead of finding the prices for each of the materials, you can take advantage of how the answer choices are set up to speed up the elimination process. Option III, soapstone, appears in three of the four choices, so check it first. Soapstone is $97 per square foot, which is over budget, so eliminate (A), (B), and (D), which all contain option III. **(C)** is correct.

43. B

Difficulty: Medium

Category: Special Right Triangles

Getting to the Answer: Use the area of one isosceles triangle, $\frac{9}{2}$, to solve for the length of the legs of the triangle and add the sum of both legs to the side lengths of the octagonal table given in the diagram. The formula for the area of a triangle is $A = \frac{1}{2}bh$, and since the base and the height are the same for an isosceles right triangle, $b = h$, you can simplify the formula: $A = \frac{b^2}{2}$. Plug in $\frac{9}{2}$ for the area and solve for the length of one of the legs:

$$A = \frac{b^2}{2}$$
$$\frac{9}{2} = \frac{b^2}{2}$$
$$b^2 = 9$$
$$b^2 = \pm 3$$

Lengths cannot be negative, so the length of each leg is 3 feet. Adding the side length to both ends of the 6-foot dimension results in $6 + 3 + 3 = 12$ feet. Adding the side length to both ends of the 4-foot dimension results in $4 + 3 + 3 = 10$ feet. The dimensions of the original piece of lumber are 10 by 12. **(B)** is correct.

Another approach is to recognize that if you use the variable x to represent the leg length of the right isosceles triangle, then the dimensions of the original piece of lumber must be $(6 + 2x)$ feet by $(4 + 2x)$ feet. By extension, since the $2x$ is the same in both, the dimensions of the original piece of lumber must only differ by $6 - 4$, or 2, feet. Eliminate (A) and (D) because the dimensions do not differ by 2 inches. Test (B) by setting $(6 + 2x)$ feet $= 12$ feet to find a value for the leg length of the triangle:

$$6 + 2x = 12$$
$$2x = 6$$
$$x = 3$$

Plugging $x = 3$ into the area formula results in $\frac{3^2}{2} = \frac{9}{2}$, which confirms **(B)** as the correct answer.

44. C

Difficulty: Medium

Category: Triangles

Getting to the Answer: To calculate the amount of leftover material for all 60 students, first calculate the amount of material that will remain when one student cuts out the pattern. Note that while the dimensions of the material are given in inches, the final answer is in square feet. Do the conversion up front: each student starts out with a 1.5 foot by 2 foot piece of material, the area of which is $1.5 \times 2 = 3 \text{ ft}^2$.

How do you find the area of the kite? Since it is symmetrical along the vertical axis, if you fold the pattern in half along the vertical axis, you have two equal triangles with a base of 2 feet and a height of $\frac{1.5}{2} = 0.75$ ft. Plugging those values into the area formula results in:

$$\text{Area of both triangles} = 2\left(\frac{1}{2}\right)(2 \text{ ft})(0.75 \text{ ft}) = 1.5 \text{ ft}^2$$

The difference between the area of the original piece of material and area of the kite, $3 - 1.5 = 1.5 \text{ ft}^2$, is the area of leftover material for one kite. To find the total amount of leftover material, multiply 1.5 ft^2 by 60 to get 90 ft^2. **(C)** is correct. If you noticed that the area of the kite is exactly half the area of the original rectangle, you can save time and simply calculate the area of the kite and multiply it by 60. If you chose (D), be careful and always make sure to check the units.

45. 6

Difficulty: Hard

Category: Special Right Triangles

Getting to the Answer: Since C is the center of the semicircle and points A, E, and D lie on the circumference, CA, CE, and CD are each equal to the radius of the semicircle. Note that CA is also the base of triangle ABC, which is a 30-60-90 triangle given that angle ABC is 30° and that angle BAC is 90°. You know $CB = 12$, so use the ratio of the sides $x{:}x\sqrt{3}{:}2x$ to find CA:

$$\frac{CA}{12} = \frac{1}{2}$$
$$CA = \frac{1}{2}(12)$$
$$CA = 6$$

Thus, $CA = 6$ and $CE = \mathbf{6}$. Grid in **6**.