

CHAPTER 7

Ratios, Proportions, and Percents

LEARNING OBJECTIVES


After completing this chapter, you will be able to:

- Set up and solve a proportion for a missing value
- Use ratios to perform unit conversions
- Calculate percents and percent change


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How Much Do You Know?


Directions: Try the questions that follow. Show your work so that you can compare your solutions to the ones found in the Check Your Work section immediately after this question set. The “Category” heading in the explanation for each question gives the title of the lesson that covers how to solve it. If you answered the question(s) for a given lesson correctly, and if your scratch-work looks like ours, you may be able to move quickly through that lesson. If you answered incorrectly or used a different approach, you may want to take your time on that lesson.

1.  Leonardo da Vinci asserted that a person’s height is proportional to the length of the person’s palm. Don’s palm is 5 centimeters in length and he is 120 centimeters tall. Mateo’s palm is 7.5 centimeters in length. Assuming that da Vinci’s theory is correct, how tall, in centimeters, is Mateo?


- A) 60 centimeters
- B) 80 centimeters
- C) 160 centimeters
- D) 180 centimeters

2.  As of 2009, there were about 225 quarters for every 2 fifty-cent coins in circulation. If there were 380 million quarters in circulation that year, approximately how many total quarters and fifty-cent coins, in millions, were there in circulation?


- A) 383
- B) 415
- C) 493
- D) 605

3.  If $\frac{5}{7a} = \frac{1}{b-a}$, which of the following proportions is equivalent?

- A) $\frac{a}{b} = \frac{5}{12}$
- B) $\frac{a}{b} = \frac{5}{2}$
- C) $\frac{b-a}{a} = \frac{5}{7}$
- D) $\frac{b-a}{a} = -\frac{7}{5}$

4.  British Thermal Units (BTUs) and calories are two measures of heat energy. If there are 2,016 calories in 8 British Thermal Units, how many calories are in 3 British Thermal Units?


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1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

5.  Sai's car is leaking engine oil at a rate of 2.5 milliliters per hour. If his car's engine contains 6 liters of oil, what will be the amount of oil, in milliliters, remaining in Sai's engine after 24 hours? (Note: There are 1,000 milliliters in 1 liter.)


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	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Category	Percentage
Completely satisfied	
Somewhat satisfied	
Somewhat dissatisfied	15%
Completely dissatisfied	25%

6. A company conducts a customer satisfaction survey. The results are partly summarized in the table. If 240 customers responded to the survey, how many customers were either completely or somewhat satisfied?
- A) 60
B) 96
C) 144
D) 204

7.  In a week, a light bulb factory produces 12,500 light bulbs. The ratio of light emitting diodes (LED bulbs) to compact fluorescent lamps (CFL bulbs) is 2:3. Of the LED bulbs produced, 3 percent were defective. If the factory produces no other kind of bulbs, how many LED bulbs were NOT defective?

- A) 2,425
B) 4,850
C) 7,275
D) 8,083

8.  In 1950, scientists estimated a certain animal population in a particular geographical area to be 6,400. In 2000, the population had risen to 7,200. If this animal population experiences the same percent increase over the next 50 years, what will the approximate population be?

- A) 8,000
B) 8,100
C) 8,400
D) 8,600

Check Your Work

1. D

Difficulty: Easy

Category: Proportions

Getting to the Answer: Since you are given the palm-to-height ratio for Don's as well as Mateo's palm length, set up a proportion using M to represent Mateo's height:

$$\frac{5 \text{ cm}}{120 \text{ cm}} = \frac{7.5 \text{ cm}}{M}$$

Cross-multiply to solve for M :

$$5M = 7.5(120)$$

$$M = \frac{7.5(120)}{5}$$

$$M = 180$$

The correct answer is **(D)**.

2. A

Difficulty: Easy

Category: Proportions

Getting to the Answer: Notice that the question asks you for the total number of coins in millions, but the ratio given specifies only the parts. Set up a proportion using the given ratio:

$$\frac{225 \text{ quarters}}{2 \text{ fifty-cent coins}} = \frac{380 \text{ million quarters}}{x \text{ fifty-cent coins}}$$

Cross-multiply: $225x = 2 \times 380$ million. Solve for x to find that $x = 3.3\bar{7}$ million fifty-cent coins in circulation. To calculate the total number of fifty-cent coins and quarters in circulation in millions in 2009, add the number of fifty-cent coins to the number of quarters: $3.3\bar{7}$ million + 380 million \approx 383 million. Choice **(A)** is correct.

3. A

Difficulty: Medium

Category: Proportions

Getting to the Answer: Since the answer choices are expressed as $\frac{a}{b} =$ and $\frac{b-a}{a} =$, cross-multiply the proportion and rewrite it to get an expression that matches one of the answer choices. Solving for $\frac{b-a}{a}$, you get the following:

$$\frac{5}{7a} = \frac{1}{b-a}$$

$$5(b-a) = 7a$$

$$\frac{b-a}{a} = \frac{7}{5}$$

This does not match (C) or (D), so eliminate them. Now solve for $\frac{a}{b}$:

$$5(b-a) = 7a$$

$$5b - 5a = 7a$$

$$5b = 12a$$

$$\frac{5}{12} = \frac{a}{b}$$

This matches **(A)**.

4. 756

Difficulty: Easy

Category: Unit Conversion

Getting to the Answer: Let x represent the number of calories in 3 BTUs, and set up a proportion equating the ratio of 8 BTUs to 2,016 calories to the ratio of 3 BTUs to x calories: $\frac{8 \text{ BTU}}{2,016 \text{ CAL}} = \frac{3 \text{ BTU}}{x \text{ CAL}}$. Solving for x gives 756 calories. Grid in **756**.

5. 5940

Difficulty: Medium**Category:** Unit Conversion

Getting to the Answer: First, determine the amount of oil that will leak out of Sai's engine after 24 hours: $\frac{2.5 \text{ mL}}{1 \text{ hr}} \times 24 \text{ hr} = 60 \text{ mL}$. Then, use the conversion to determine the amount of oil, in milliliters, initially in the engine: $6 \cancel{\text{ L}} \times \frac{1,000 \text{ mL}}{1 \cancel{\text{ L}}} = 6,000 \text{ mL}$. Calculate the difference between the two to find the amount of oil remaining in the engine after 24 hours: $6,000 \text{ mL} - 60 \text{ mL} = 5,940 \text{ mL}$. Grid in **5940**.

6. C

Difficulty: Easy**Category:** Percents

Getting to the Answer: According to the table, 25% of the customers are completely dissatisfied and 15% are somewhat dissatisfied. So, $100\% - 25\% - 15\% = 60\%$ of the customers are completely or somewhat satisfied. Thus, the number of customers who are completely or somewhat satisfied is 60% of 240, or $0.6 \times 240 = 144$ customers, making **(C)** correct.

7. B

Difficulty: Hard**Category:** Percents

Getting to the Answer: Approach the question as a series of steps. Before you bubble in your answer, check that you answered the right question (the number of LED bulbs that were *not* defective).

First, find the total number of LED bulbs produced. Since the ratio of LED to CFL is 2:3 and a total of 12,500 of both kinds of bulbs were produced, two parts LED plus three parts CFL equals 12,500. Let x be the number of LED bulbs produced and use the part-to-whole ratio to set up a proportion:

$$\frac{2}{5} = \frac{x}{12,500}$$

Solve for x by cross-multiplying:

$$\begin{aligned} 2(12,500) &= 5x \\ 25,000 &= 5x \\ 5,000 &= x \end{aligned}$$

Now find the number of LED bulbs that were not defective: $5,000 \times 97\% = 5,000 \times 0.97 = 4,850$. Choice **(B)** is correct.

8. B

Difficulty: Medium**Category:** Percent Change

Getting to the Answer: Find the percent increase in the population from 1950 to 2000 using this formula: percent increase = $\frac{\text{amount of increase}}{\text{original amount}} \times 100\%$. Then apply the same percent increase to the animal population in 2000.

The amount of increase is $7,200 - 6,400 = 800$, so the percent increase is

$\frac{800}{6,400} \times 100\% = 0.125 \times 100\% = 12.5\%$ between 1950 and 2000. If the total percent increase over the next 50 years is the same, the animal population should be about

$$\begin{aligned} 7,200 \times (100\% + 12.5\%) &= 7,200 \times (112.5\%) = \\ &7,200 \times 1.125 = 8,100. \end{aligned}$$

(B) is correct.

Ratios and Proportions

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Set up and solve a proportion for a missing value

To answer a question like this:



The World War II aircraft carrier *Essex* was 872 feet long with a beam (width) of 147 feet. A museum wishes to build an exact replica scale model of the *Essex* that is 8 feet long. Approximately how many inches wide will the scale model's beam be? (1 foot = 12 inches)

- A) 13
- B) 16
- C) 26
- D) 109

You need to know this:

A **ratio** is a comparison of one quantity to another. When writing ratios, you can compare one part of a group to another part of that group or you can compare a part of the group to the whole group. Suppose you have a bowl of apples and oranges: you can write ratios that compare apples to oranges (part to part), apples to total fruit (part to whole), and oranges to total fruit (part to whole).

Keep in mind that ratios convey *relative* amounts, not necessarily actual amounts, and that they are typically expressed in lowest terms. For example, if there are 10 apples and 6 oranges in a bowl, the ratio of apples to oranges would likely be expressed as $\frac{5}{3}$ on the SAT rather than as $\frac{10}{6}$. However, if you know the ratio of apples to oranges and either the actual number of apples or the total number of pieces of fruit, you can find the actual number of oranges by setting up a proportion (see below).

Note that the SAT may occasionally use the word “proportion” to mean “ratio.”

A **proportion** is simply two ratios set equal to each other, such as $\frac{a}{b} = \frac{c}{d}$. Proportions are an efficient way to solve certain problems, but you must exercise caution when setting them up. Noting the units of each piece of the proportion will help you put each piece of the proportion in the right place. Sometimes the SAT may ask you to determine whether certain proportions are equivalent—check this by cross-multiplying. You'll get results that are much easier to compare.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then: } ad = bc, \frac{a}{c} = \frac{b}{d}, \frac{d}{b} = \frac{c}{a}, \frac{b}{a} = \frac{d}{c}, \text{ BUT } \frac{a}{d} \neq \frac{c}{b}$$

Each derived ratio shown except the last one is simply a manipulation of the first, so all except the last are correct. You can verify this via cross-multiplication ($ad = bc$ in each case except the last).

Alternatively, you can pick equivalent fractions $\frac{2}{3}$ and $\frac{6}{9}$ ($a = 2, b = 3, c = 6, d = 9$). Cross-multiplication gives $2 \times 9 = 3 \times 6$, which is a true statement. Dividing 2 and 3 by 6 and 9 gives $\frac{2}{6} = \frac{3}{9}$, which is also true, and so on. However, attempting to equate $\frac{2}{9}$ and $\frac{3}{6}$ will not work.

If you know any three numerical values in a proportion, you can solve for the fourth. For example, say a fruit stand sells 3 peaches for every 5 apricots, and you are supposed to calculate the number of peaches sold on a day when 20 apricots were sold. You would use the given information to set up a proportion and solve for the unknown:

$$\frac{3}{5} = \frac{p}{20}$$

You can now solve for the number of peaches sold, p , by cross-multiplying:

$$\begin{aligned} 60 &= 5p \\ p &= 12 \end{aligned}$$

Alternatively, you could use the common multiplier to solve for p : the numerator and denominator in the original ratio must be multiplied by the same value to arrive at their respective terms in the new ratio. To get from 5 to 20 in the denominator, you multiply by 4, so you also have to multiply the 3 in the numerator by 4 to arrive at the actual number of peaches sold: $4(3) = 12$.

You need to do this:

Set up a proportion and solve for the unknown, either by cross-multiplying or by using the common multiplier.

Explanation:

The ratio of the length of the real *Essex* to that of the scale model is $\frac{872 \text{ ft}}{8 \text{ ft}}$. You know the actual beam width (147 feet), so set up a proportion and solve for the scale model's beam width:

$$\begin{aligned} \frac{872 \text{ ft}}{8 \text{ ft}} &= \frac{147 \text{ ft}}{x \text{ ft}} \\ 872x &= 1,176 \text{ ft} \\ x &\approx 1.349 \text{ ft} \end{aligned}$$

The question asks for the answer in inches, not feet, so multiply by 12 inches per foot: $1.349 \text{ ft} \times 12 \text{ in/ft} = 16.188$ inches. The correct answer is **(B)**.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

1. Teachers at a certain school know that, when reviewing for exams, the number of topics they can cover is directly proportional to the length of time they have to review. If teachers can cover 9 topics in a single 45-minute period, how many topics can they cover in a 1-hour period?



- A) 5
- B) 7
- C) 10
- D) 12

2. Objects weigh less on the Moon because the Moon's gravitational pull is less than Earth's. In general, 1 pound on Earth is equal to approximately 0.166 pounds on the Moon. If a person weighs 29 pounds on the Moon, approximately how much, in pounds, does the person weigh on Earth?



- A) 21
- B) 48
- C) 175
- D) 196

3. A machine produces 6 defective parts out of every 3,500 it makes. How many total parts were made during the time the machine produced 27 defective parts?



- A) 14,000
- B) 15,750
- C) 17,500
- D) 21,000

4. The ratio of freshmen to sophomores in an auditorium was 3 to 10. After an additional 270 freshmen and 120 sophomores entered the auditorium, the ratio of freshmen to sophomores was 6 to 5. No other students entered or left the auditorium. How many freshmen were in the auditorium before the additional students entered?



- A) 15
- B) 42
- C) 140
- D) 182

HINT: For Q5, which conversion will be easier? Minutes to hours or hours to minutes?

5. Riding her bicycle, Reyna can travel 1 mile in 5.5 minutes. If she rides at a constant rate, which of the following is closest to the distance she will travel in 1.5 hours?



- A) 9 miles
- B) 11 miles
- C) 13 miles
- D) 16 miles

6. If $\frac{x+y}{x} = \frac{4}{9}$, which of the following proportions is equivalent?



- A) $\frac{y}{x} = -\frac{5}{9}$
- B) $\frac{y}{x} = \frac{13}{9}$
- C) $\frac{y-x}{x} = -\frac{4}{9}$
- D) $\frac{y-x}{x} = -\frac{9}{4}$

HINT: For Q7, start with the proportion $\frac{\text{physicists}}{\text{total}} = \frac{2}{5}$, then think about what to substitute for “physicists” and “total.”

7. All of the attendees at a symposium are either physicists or biologists. If there are 123 physicists and 270 biologists, then how many additional physicists must arrive at the symposium in order for the ratio of physicists to total attendees to become 2 to 5?



- A) 25
- B) 50
- C) 57
- D) 114

Unit Conversion

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Use ratios to perform unit conversions

To answer a question like this:



The nearest star to the Sun, Proxima Centauri, is approximately 4.3 light-years away. Another star, Sirius A, is twice that distance from the Sun. If 1 light-year equals 63,000 astronomical units (AU), and 1 AU equals 150 million kilometers, approximately how far is Sirius A from the Sun in trillions of kilometers? (1 trillion = 1,000,000,000,000)

- A) 2.2
- B) 20
- C) 41
- D) 81

You need to know this:

You can use ratios to perform unit conversions. This is especially useful when there are multiple conversions or when the units are unfamiliar.

For example, though these units of measurement are no longer commonly used, there are 8 furlongs in a mile and 3 miles in a league. Say you're asked to convert 4 leagues to furlongs. A convenient way to do this is to set up the conversion ratios so that equivalent units cancel:

$$4 \text{ leagues} \times \frac{3 \text{ miles}}{1 \text{ league}} \times \frac{8 \text{ furlongs}}{1 \text{ mile}} = 4 \times 3 \times 8 = 96 \text{ furlongs}$$

Notice that all the units cancel out except the furlongs, which is the one you want.

You need to do this:

Set up a series of ratios to make equivalent units cancel. (Keep track of the units by writing them down next to the numbers in the ratios.) You should be left with the units you're converting into.

Explanation:

Sirius A is twice as far from the Sun as Proxima Centauri, so it is $2(4.3) = 8.6$ light-years away from the Sun. Set up a series of ratios to convert to trillion kilometers:

$$\begin{aligned} 8.6 \cancel{\text{light-years}} \times \frac{63,000 \cancel{\text{AU}}}{1 \cancel{\text{light-year}}} \times \frac{150 \text{ million km}}{1 \cancel{\text{AU}}} &= 8.6 \times 63,000 \times 150 \text{ million km} \\ &= 81,270,000 \text{ million km} \\ &= 81.27 \text{ trillion km} \end{aligned}$$

Because there are 6 zeros in a million, 81,270,000 million is 81,270,000,000,000. There are 12 zeros in a trillion, so this number equals 81.27 trillion. The correct answer is **(D)**.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q8, *cubic feet* means ft^3 or $\text{ft} \times \text{ft} \times \text{ft}$.

8. Quinn wants to rent a self-storage unit for her college dorm room furniture for the summer. She estimates that she will need 700 cubic feet of storage space, but the self-storage provider measures its units in cubic meters. If 1 meter is approximately 3.28 feet, about how many cubic meters of space will Quinn need?



- A) 19.84
- B) 25.93
- C) 65.07
- D) 213.41

9. Because court reporters must type every word at a trial or hearing, they must be able to type at a minimum rate of 3.75 words per second in order to be certified. Suppose a trial transcript contains 25 pages with an average of 675 words per page. Assuming the court reporter typed the transcript at the minimum rate, how long was he actively typing?



- A) 1 hour, 15 minutes
- B) 1 hour, 45 minutes
- C) 2 hours, 30 minutes
- D) 3 hours

HINT: For Q10, “how many more” means you’re solving for a difference. Subtract, then convert pounds/hour to ounces/minute.

10. At 350°F , an oven can cook approximately 3 pounds of turkey per hour. At 450°F , it can cook approximately 4.5 pounds per hour. How many more ounces of turkey can the oven cook at 450°F than at 350°F in 10 minutes? (1 pound = 16 ounces)



- A) 4
- B) 6
- C) 8
- D) 12

11. An emergency room doctor prescribes a certain pain medication to be delivered through an IV drip. She prescribes 800 milliliters of the medication to be delivered over the course of 8 hours. The IV delivers 1 milliliter of medication over the course of 30 drips. How many drips per minute are needed to deliver the prescribed dosage?



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2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12. Botanists studying a particular coastal redwood tree determined that the tree grew 46 meters in the first 50 years of its life. On average, how many centimeters per day did it grow during this period? Assume that there are 365 days in a year, and round your answer to the nearest hundredth of a centimeter. (1 meter = 100 centimeters)



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	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Percents

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Calculate percents

To answer a question like this:



Political canvassers polled voters in two locations on whether they viewed a particular candidate for governor favorably. At the first location, they asked 125 people and of those, 22.4 percent responded favorably. At the second location, 37.5 percent of 272 people responded favorably. Approximately what percent of all the people surveyed responded favorably?

- A) 25.7%
- B) 30.0%
- C) 31.5%
- D) 32.7%

You need to know this:

To calculate percents, use this basic equation:

$$\text{Percent} = \frac{\text{part}}{\text{whole}} \times 100\%$$

Alternatively, use this statement: [blank] percent of [blank] is [blank]. Translating from English into math, you get $[\text{blank}]\% \times [\text{blank}] = [\text{blank}]$.

You need to do this:

- Plug in the values for any two parts of the formula and solve for the third.
- In some calculations, you may find it convenient to express percents as decimals. To do this, use the formula above but stop before you multiply by 100% at the end.

Explanation:

Use a variation of the three-part percent formula to answer this question: whole \times percent = part, where the percent is expressed as a decimal.

First, find the number of people at each location who responded favorably using the formula. Start with the first location: $125 \times 0.224 = 28$. Move on to the second location: $272 \times 0.375 = 102$. Next, find the total number of people who were surveyed at both locations, which is $125 + 272 = 397$, and the total number who responded favorably, $28 + 102 = 130$. Finally, find the percent of people who responded favorably by using the formula one more time:

$$\begin{aligned} 397 \times \text{percent} &= 130 \times 100\% \\ \text{percent} &= \frac{130}{397} \times 100\% \\ &\approx 0.327 \times 100\% \\ &= 32.7\% \end{aligned}$$

Of all the people surveyed, about 32.7% responded favorably, making **(D)** the correct answer.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

13. A college athletics program found that approximately 3 percent of 308 runners were injured during workouts and that approximately 6 percent of 237 weight lifters were injured during workouts. Which of the following is the closest to the total number of runners and weight lifters who were injured?
- A) 50
B) 39
C) 26
D) 23

HINT: For Q14, what percent of the attendees are teachers?

14. At a high school conference, 15 percent of the attendees are sophomores, 30 percent are juniors, 25 percent are seniors, and the remaining 18 attendees are teachers. How many more juniors are there than seniors?

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	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

HINT: For Q15, how many gallons of *pigment* is the painter starting with? How many gallons of *pigment* are needed for the final mix? How many gallons of the final paint will it take to provide the needed pigment?

15. A painter has 20 gallons of a paint mixture that is 15 percent blue pigment. How many gallons of a mixture that is 40 percent blue pigment would the painter need to add to achieve a mixture that is 20 percent blue pigment?
- A) 4
B) 5
C) 8
D) 12
16. On August 1, the price of one share of a company's stock was \$75. On September 1, the price of one share was \$10 more than it was on August 1 and 80 percent of the price of one share on October 1. To the nearest dollar, what was the price of one share on October 1?
- A) \$68
B) \$99
C) \$102
D) \$106



Percent Change

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Calculate percent change

To answer a question like this:



On a particular day, a power company makes several changes in the power allocated to a neighborhood. First, it increases the power by 20 percent. Then, it decreases the power by 10 percent. Finally, it increases the power by 30 percent. What is the net percent increase in this neighborhood's power allocation, to the nearest tenth of a percent? (The percent sign is understood after your answer. For example, if the answer is 15.1%, grid in 15.1.)

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	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

You need to know this:

You can determine the **percent change** in a given situation by applying this formula:

$$\text{Percent increase or decrease} = \frac{\text{amount of increase or decrease}}{\text{original amount}} \times 100\%$$

Sometimes, more than one change will occur. Be careful here, as it can be tempting to take a shortcut by just adding two percent changes together (which will almost always lead to an incorrect answer). Instead, you'll need to find the total amount of the increase or decrease and then apply the formula.

You need to do this:

- Calculate the actual increase or decrease.
- Divide by the *original* amount (not the new amount!).
- Multiply by 100%.

Explanation:

The question does not give an initial value for power allocation, so pick 100 (often the best number to use when picking numbers for questions involving percents) and then calculate the actual change. A 20% increase from 100 is $100 + 100 \times 0.2$ and brings the power allocation to $100 + 20 = 120$. A 10% decrease from 120 is $120 - 120 \times 0.1$ and brings the power allocation to $120 - 12 = 108$. Lastly, an increase of 30% puts the final power allocation at $108 + 0.3 \times 108 = 108 + 32.4 = 140.4$. The actual increase, then, is $140.4 - 100 = 40.4$. (Again, note that simply combining the percents would get you the wrong answer: $10\% - 20\% + 30\% = 40\%$.)

Plugging this increase into the percent change formula yields the following (remember to divide by the *original* amount, 100, rather than by the new amount, 140.4):


$$\text{Percent change} = \frac{40.4}{100} \times 100\% = 40.4\%$$

Grid in **40.4**.


Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q17, remember to divide by the *original* value.


17.  A homeowner's annual property tax payment was \$1,494. Due to a property value reassessment, the tax payment was increased to \$1,572. To the nearest tenth of a percent, by what percent was the homeowner's property tax payment increased?

- A) 0.1%
- B) 5.0%
- C) 5.2%
- D) 7.9%

18.  The price of a single share of a certain corporation's stock was \$35. Six months later, the price of a single share of the corporation's stock had risen to \$49. To the nearest percent, what was the percent increase in the price per share?


- A) 14%
- B) 29%
- C) 40%
- D) 48%

HINT: For Q19, how does the wording of the question help you determine which container of coins is the original amount?

19.  The number of coins in jar X is 75. The number of coins in jar Y is 54. By what percent is the number of coins in jar Y less than the number of coins in jar X?


- A) 21%
- B) 28%
- C) 39%
- D) 72%

HINT: For Q20, if you have 75% more senior than juniors, you have all the juniors (100%) plus 75%, or 175%. Adding the percents at the start saves a calculation step.

20.  At a school rally, there are 50 sophomores, 80 juniors, and 75 percent more seniors than juniors. By what percent is the number of seniors greater than the number of sophomores?

- A) 80%
- B) 140%
- C) 150%
- D) 180%

HINT: For Q21, the final 25% discount is applied to an already reduced price. You *cannot* add the percent discounts together.

21.  The original price of a newly released smart phone was y dollars. A year later, the original price of the phone was discounted by 36 percent. After another six months, an online retailer was selling the phone at a price that was 25 percent less than the previously discounted price. By what percent was the online retailer's price less than y ?

- A) 27%
- B) 48%
- C) 52%
- D) 61%

On Test Day

When a question features multiple percentages, you have to make a key strategic decision: can I do the arithmetic on the percentages themselves and get the answer right away, or do I have to calculate each percentage individually and do the arithmetic on the actual values?

For example, suppose a car traveling 50 miles per hour increases its speed by 20 percent and then decreases its speed by 20 percent. Can you just say that its final speed is 50 miles per hour since $+20\% - 20\% = 0$? No, because after a 20% increase, the car's speed becomes 120% of the original: $1.2(50) = 60$. When the car "decreases its speed by 20 percent," that 20 percent is calculated based on the new speed, 60, not the original speed, and 20 percent of 60 is greater than 20 percent of 50. Thus, the car's final speed is lower than its starting speed: $50(1.2)(0.8) = 48$ miles per hour.

By contrast, suppose you have to find how many more nonsmokers than occasional smokers live in a certain region where there are 13,450 residents, given that 62 percent of them don't smoke and 8 percent of them do smoke occasionally. It may be tempting to find 62 percent of 13,450 ($0.62 \times 13,450 = 8,339$), then find 8 percent of 13,450 ($0.08 \times 13,450 = 1,076$), and finally subtract those two numbers to get the answer ($8,339 - 1,076 = 7,263$). This is a waste of time, though. Instead, you can quickly find the difference between the two percentages ($62 - 8 = 54$) and take 54 percent of the total to get the answer in one step: $13,450 \times 0.54 = 7,263$, the same answer.

If you *can* do arithmetic using the percentages but choose to do arithmetic on the raw numbers instead, you'll waste time doing unnecessary work. But if you *can't* do arithmetic on the percentages (as in the first example) but do anyway, then you'll get the wrong answer. So, being able to tell whether you can or can't do the arithmetic on the percentages is a useful skill.

Luckily, the fundamental principle is simple: you can always do arithmetic on the percentages as long as the percentages are out of the same total. If the totals are different, then you must convert the percentages into actual values. Practice applying this principle on the following question.

22. Flanders Corporation has 250 full-time and 250 part-time employees. If 92 percent of the full-time employees qualify for health insurance benefits, and 74 percent of the part-time employees do not qualify for health insurance benefits, then how many more full-time than part-time employees at Flanders Corporation qualify for health insurance benefits?
- A) 45
B) 90
C) 165
D) 330

The answer and explanation can be found at the end of this chapter.

How Much Have You Learned?

Directions: For testlike practice, give yourself 15 minutes to complete this question set. Be sure to study the explanations, even for questions you got right. They can be found at the end of this chapter.

Undergraduate Costs at a State University

2014–15	2015–16	2016–17	2017–18	2018–19	2019–20
\$12,192	\$12,804	\$13,446	\$14,118	\$14,820	\$15,564

23. The table above summarizes the total cost per undergraduate student per year at a state university for each academic year from 2014–15 to 2019–20.



If fees account for 8.75 percent of one year's total costs, what is the average fee increase per academic year? Round your answer to the nearest dollar.

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

24. A gardener planted a 20-inch-tall sapling in his yard. Four months later, the sapling was 27 inches tall. By what percent did the height of the sapling increase over the four months?
- A) 7%
B) 26%
C) 35%
D) 74%

25. From 1997 to 1998, company T's profits increased by 25 percent. From 1998 to 1999, company T's profits rose from \$375 million to \$483 million. By what percent did company T's profits increase from 1997 to 1999?



- A) 38%
B) 49%
C) 54%
D) 61%

26. At a certain store, the price of a calculator is \$150, the price of a radio is \$75, and the price of a printer is 16 percent less than the price of a radio. By what percent is the price of a printer less than the price of a calculator?



- A) 39%
B) 58%
C) 63%
D) 87%

27. A car that is traveling at a constant speed of 9 miles per hour is traveling at a constant speed of how many feet per second? (1 mile = 5,280 feet)



- A) 1.5
B) 6.1
C) 13.2
D) 79.2

28. In a certain music store, every guitar is either a dreadnought or a parlor guitar. The ratio of dreadnoughts to parlor guitars in the music store is 4 to 15, and there is a total of 114 guitars in the music store. How many guitars in the music store are dreadnoughts?



- A) 24
- B) 29
- C) 34
- D) 46

29. The population of a town was 84,600 on January 1, 2016, and 74,025 on January 1, 2017. By what percent did the population of the town decrease from January 1, 2016, to January 1, 2017?



- A) 10.5%
- B) 12.5%
- C) 14.5%
- D) 17%

30. The perimeter of regular pentagon P is half the perimeter of regular hexagon H . What is the ratio of the length of a side of the pentagon to a side of the hexagon?

- A) 1:2
- B) 3:5
- C) 5:6
- D) 5:3

31. A jar contains red, white, and yellow candy pieces in the ratio of 9:5:4, respectively. When 7 pieces of red candy and 5 pieces of white candy are removed from the jar and 3 pieces of yellow candy are added, the ratio of red to white to yellow becomes 4:2:3. If the jar contains only these three colors of candy, how many pieces were originally in the jar?



	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

32. Juan's air mattress deflates at a constant rate of 100 milliliters per minute. If Juan's air mattress contains 300 liters of air, how long will it take, in hours, for the air mattress to completely deflate? (Note: There are 1,000 milliliters in 1 liter.)

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Reflect

Directions: Take a few minutes to recall what you've learned and what you've been practicing in this chapter. Consider the following questions, jot down your best answer for each one, and then compare your reflections to the expert responses on the following page. Use your level of confidence to determine what to do next.

What is a ratio and how is it different from a proportion?

If you're given a ratio of one quantity to another, what can you say about the total number of quantities?

When doing unit conversions, how can you make sure you're doing them correctly?

Suppose the value of something increases by 20 percent. How can you calculate the final value in the fewest number of steps? What if the value decreases by 20 percent?

What is the percent change formula and what is the biggest pitfall to avoid when using it?

Expert Responses

What is a ratio and how is it different from a proportion?

A ratio is the relative comparison of one quantity to another. For example, if the ratio of dogs to cats in an animal shelter is 3 to 5, then there are 3 dogs for every 5 cats. A proportion is two ratios set equal to each other.

If you're given a ratio of one quantity to another, what can you say about the total number of quantities?

Given a ratio, you know that the total must be a multiple of the sum of the ratio's parts. For example, if the ratio of dogs to cats is 3 to 5, then the total number of dogs and cats must be a multiple of $3 + 5$, or 8. This means that when the SAT gives you one ratio, it's actually giving you several. If you're told that $\text{dogs:cats} = 3:5$, then you also know that $\text{dogs:total} = 3:8$ and $\text{cats:total} = 5:8$. You can use this "hidden" knowledge to your advantage.

When doing unit conversions, how can you make sure you're doing them correctly?

To do unit conversions correctly, set up the conversion in whichever way makes units cancel. For example, to convert 3 feet into inches, you multiply 3 feet by 12 inches per foot, because it cancels out the feet unit. If instead you multiplied 3 feet by 1 foot per 12 inches, then the resulting units would be "feet squared per inch," which makes no sense.

Suppose the value of something increases by 20 percent. How can you calculate the final value in the fewest number of steps? What if the value decreases by 20 percent?

The fastest way to increase a value by 20 percent is to multiply it by 1.2, which is $100\% + 20\% = 120\%$. Similarly, to decrease something by 20 percent, you multiply it by 0.8, as that is $100\% - 20\% = 80\%$.

What is the percent change formula and what is the biggest pitfall to avoid when using it?

The percent change formula is as follows:

$$\text{Percent change} = \frac{\text{amount of increase or decrease}}{\text{original amount}} \times 100\%$$

A common mistake is to put the new amount on the bottom of the fraction rather than the original amount.

Next Steps

If you answered most questions correctly in the "How Much Have You Learned?" section, and if your responses to the Reflect questions were similar to those of the SAT expert, then consider ratios and the related topics in this chapter to be an area of strength and move on to the next chapter. Come back to this topic periodically to prevent yourself from getting rusty.

If you don't yet feel confident, review those parts of this chapter that you have not yet mastered and try the questions you missed again. As always, be sure to review the explanations closely. Finally, **go online** (www.kaptest.com/moreonline) for additional practice on the highest yield topics in this chapter.

Answers and Explanations

1. D

Difficulty: Easy

Getting to the Answer: To answer a question that says “directly proportional,” set two ratios equal to each other and solve for the missing amount. Be sure to match the units in the numerators and in the denominators on both sides of the proportion.

Because the first rate is given in minutes, write 1 hour as 60 minutes. Let t equal the number of topics the teachers can cover in a 60-minute period. Set up a proportion and solve for t :

$$\begin{aligned}\frac{9 \text{ topics}}{45 \text{ minutes}} &= \frac{t \text{ topics}}{60 \text{ minutes}} \\ 9(60) &= 45(t) \\ 540 &= 45t \\ 12 &= t\end{aligned}$$

Choice **(D)** is correct.

2. C

Difficulty: Easy

Getting to the Answer: Think about how your answer should look. A person weighs *less* on the Moon, so that person should weigh *more* on Earth. This means your answer must be greater than 29, so you can eliminate (A) right away.

Now, set up a proportion:

$$\begin{aligned}\frac{0.166 \text{ lb on Moon}}{1 \text{ lb on Earth}} &= \frac{29 \text{ lb on Moon}}{p \text{ lb on Earth}} \\ 0.166p &= 29(1) \\ p &\approx 174.7\end{aligned}$$

The person weighs about 175 pounds on Earth. Choice **(C)** is correct.

3. B

Difficulty: Easy

Getting to the Answer: This is a typical proportion question. Use words first to write the proportion. Then translate from English into math. Let n equal the total number of parts made. Set up a proportion and solve for n . Be sure to match the units in the numerators and in the denominators on both sides of the proportion:

$$\begin{aligned}\frac{\text{defective parts}}{\text{number made}} &= \frac{\text{defective parts}}{\text{number made}} \\ \frac{6}{3,500} &= \frac{27}{n} \\ 6n &= 27(3,500) \\ 6n &= 94,500 \\ n &= 15,750\end{aligned}$$

This means **(B)** is correct.

4. B

Difficulty: Hard

Getting to the Answer: This question has two unknowns: you don’t know the starting number of either freshmen or sophomores. To solve for two unknowns, you need two equations. Let f represent the original number of freshmen in the auditorium and s represent the original number of sophomores. The starting ratio is $\frac{f}{s} = \frac{3}{10}$. Cross-multiplying yields $10f = 3s$. This is your first equation.

Set up a second equation to represent the adjusted number of freshmen and sophomores:

$$\begin{aligned}\frac{f + 270}{s + 120} &= \frac{6}{5} \\ 5(f + 270) &= 6(s + 120) \\ 5f + 1,350 &= 6s + 720\end{aligned}$$

You’ve determined from the first ratio that $10f = 3s$, and if you multiply this equation by 2, you get $20f = 6s$. Now substitute $20f$ for $6s$ in the above equation:

$$\begin{aligned}5f + 1,350 &= 20f + 720 \\ 630 &= 15f \\ 42 &= f\end{aligned}$$

There were 42 freshmen in the auditorium to start, so **(B)** is correct.

5. D

Difficulty: Medium

Getting to the Answer: Use the known time of 5.5 minutes it takes Reyna to travel 1 mile to calculate the distance she can cover in 1.5 hours. So that you're working with the same units, first convert 1.5 hours to minutes: $1.5 \times 60 = 90$ minutes. Let d be the unknown distance and then set up a proportion to solve for d :

$$\begin{aligned}\frac{1}{5.5} &= \frac{d}{90} \\ 90 &= 5.5d \\ \frac{90}{5.5} &= d \\ d &\approx 16\end{aligned}$$

Therefore, **(D)** is correct.

6. A

Difficulty: Medium

Getting to the Answer: Since the answer choices are expressed as $\frac{y}{x} =$ and $\frac{y-x}{x} =$, cross-multiply the proportion and rewrite it to get an expression that matches the form of one of the answer choices. Solve for $\frac{y}{x}$:

$$\begin{aligned}\frac{x+y}{x} &= \frac{4}{9} \\ 9(x+y) &= 4x \\ 9x+9y &= 4x \\ 9y &= -5x \\ \frac{y}{x} &= -\frac{5}{9}\end{aligned}$$

This matches **(A)**.

Alternatively, you could rewrite the proportion as follows:

$$\begin{aligned}\frac{x}{x} + \frac{y}{x} &= \frac{4}{9} \\ 1 + \frac{y}{x} &= \frac{4}{9} \\ \frac{y}{x} &= \frac{4}{9} - 1 \\ \frac{y}{x} &= -\frac{5}{9}\end{aligned}$$

Note that multiplying both sides of the proportion by -1 would give $-\frac{(x+y)}{x} = -\frac{4}{9}$ or $\frac{-x-y}{x} = -\frac{4}{9}$, which does not match (C) or (D).

7. C

Difficulty: Hard

Getting to the Answer: The ratio of physicists to total attendees is the number of physicists divided by the number of all attendees. Suppose x new physicists arrive at the symposium. The new number of physicists will be $123 + x$, and the new number of all attendees will be the original physicists (123) + biologists (270) + the newcomer physicists (x). The ratio of the first number over the second equals 2 to 5 , so set up a proportion and solve for x :

$$\begin{aligned}\frac{123+x}{123+270+x} &= \frac{2}{5} \\ \frac{123+x}{393+x} &= \frac{2}{5} \\ 5(123+x) &= 2(393+x) \\ 615+5x &= 786+2x \\ 3x &= 171 \\ x &= 57\end{aligned}$$

Therefore, **(C)** is correct.

8. A

Difficulty: Medium

Getting to the Answer: Map out your route from starting units to ending units, being mindful of the fact that the question deals with units of volume (cubic units). The starting quantity is in ft^3 , and the desired quantity is in m^3 . The only conversion factor you need is $1 \text{ m} \approx 3.28 \text{ ft}$, but you'll need to use it three times. Setting up your route to m^3 , you get:

$$\begin{aligned}\frac{700 \text{ ft}^3}{1} \times \frac{1 \text{ m}}{3.28 \text{ ft}} \times \frac{1 \text{ m}}{3.28 \text{ ft}} \times \frac{1 \text{ m}}{3.28 \text{ ft}} &= \frac{700}{(3.28)^3} \text{ m}^3 \\ &\approx 19.84 \text{ m}^3\end{aligned}$$

This matches **(A)**.

9. A

Difficulty: Hard

Getting to the Answer: Whenever multiple rates are given, pay very careful attention to the units. Starting with the number of pages the reporter typed, set up your conversion ratios so that equivalent units cancel. Be sure your units match those in the answer choices:

$$25 \cancel{\text{ pages}} \times \frac{675 \cancel{\text{ words}}}{1 \cancel{\text{ page}}} \times \frac{1 \cancel{\text{ second}}}{3.75 \cancel{\text{ words}}} \\ \times \frac{1 \cancel{\text{ minute}}}{60 \cancel{\text{ seconds}}} \times \frac{1 \text{ hour}}{60 \cancel{\text{ minutes}}} = 1.25 \text{ hours}$$

Because 1.25 hours is not an answer choice, convert 0.25 to minutes: $0.25 \times 60 \text{ minutes} = 15 \text{ minutes}$, making (A) the correct answer.

10. A

Difficulty: Medium

Getting to the Answer: The 450 °F oven cooks $4.5 - 3 = 1.5$ more pounds per hour than the 350 °F oven. However, the question asks for the answer in ounces per 10 minutes, so convert from pounds per hour to ounces per minute, then multiply by 10 minutes:

$$\frac{1.5 \cancel{\text{ lb}}}{1 \cancel{\text{ hr}}} \times \frac{16 \cancel{\text{ oz}}}{1 \cancel{\text{ lb}}} \times \frac{1 \cancel{\text{ hr}}}{60 \cancel{\text{ min}}} \times 10 \text{ min} = 4 \text{ oz}$$

In 10 minutes, the oven at 450 °F can cook 4 ounces more than the oven at 350 °F, making (A) the correct answer.

11. 50

Difficulty: Medium

Getting to the Answer: Starting with the prescribed dosage, set up your conversion ratios so that equivalent units cancel and you get drips per minute:

$$\frac{800 \cancel{\text{ mL}}}{8 \cancel{\text{ hours}}} \times \frac{30 \cancel{\text{ drips}}}{1 \cancel{\text{ mL}}} \times \frac{1 \cancel{\text{ hour}}}{60 \cancel{\text{ minutes}}} = 50 \frac{\text{drips}}{\text{minute}}$$

Grid in 50.

12. 0.25

Difficulty: Easy

Getting to the Answer: The question provides the growth rate, in meters, over a 50-year period. You need to convert this to a rate of centimeters per day. Set up your conversion ratios to make the units cancel:

$$\frac{46 \cancel{\text{ meters}}}{50 \cancel{\text{ years}}} \times \frac{100 \cancel{\text{ centimeters}}}{1 \cancel{\text{ meter}}} \times \frac{1 \cancel{\text{ year}}}{365 \text{ days}} \\ \approx 0.252 \frac{\text{centimeters}}{\text{day}}$$

You're told to round to the nearest hundredth of a centimeter, so grid in 0.25.

13. D

Difficulty: Easy

Getting to the Answer: The question asks for the approximate combined number of runners and weight lifters who were injured. Calculate the approximate number from each group who were injured and then add the numbers together:

$$3\% \times 308 = 0.03 \times 308 \approx 9 \\ 6\% \times 237 = 0.06 \times 237 \approx 14 \\ 9 + 14 = 23$$

Therefore, (D) is correct.

14. 3

Difficulty: Medium

Strategic Advice: First find the total number of attendees, then calculate the difference in the actual number of juniors and seniors.

Getting to the Answer: The question gives you the percents of sophomores, juniors, and seniors attending the conference, as well as the actual number of teachers. Add up all of the percents to find the total percent of the attendees who are *not* teachers: $15\% + 25\% + 30\% = 70\%$. Therefore, the 18 teachers account for $100\% - 70\% = 30\%$ of the attendees.

You can solve for the total number of attendees (the whole) by plugging in the corresponding values for percent and part into the equation $\text{part} = \text{percent} \times \text{whole}$. Say the total number of attendees is t :

$$\begin{aligned} 18 &= 0.30t \\ \frac{18}{0.30} &= t \\ t &= 60 \end{aligned}$$

So the total number of attendees is 60. Juniors are 30% of this number and seniors are 25%, so the difference between juniors and seniors is $30\% - 25\% = 5\%$. Now calculate 5% of the total: $60 \times 0.05 = 3$. Thus, there are 3 more juniors than seniors. Grid in **3**.

15. B**Difficulty:** Hard

Getting to the Answer: The question gives you the amount of 15% mixture and the desired concentration (20%) that the painter wants to achieve by adding an unknown quantity of 40% mixture. In effect, the question is asking you to calculate a weighted average where the desired average is known.

Let x represent the unknown number of gallons of the 40% mixture that the painter needs to add. Set up a weighted average equation using the known amount (20 gallons) of the 15% mixture and the unknown amount of the 40% mixture to equal the desired mixture concentration of 20%:

$$\frac{0.15(20) + 0.40(x)}{20 + x} = 0.20$$

Next, multiply both sides of the equation by the denominator and then solve for x :

$$\begin{aligned} 0.15(20) + 0.40x &= 0.20(20 + x) \\ 3 + 0.40x &= 4 + 0.20x \\ 0.20x &= 1 \\ x &= 5 \end{aligned}$$

Thus, the painter needs to add 5 gallons of the 40% mixture to achieve the desired 20% concentration of blue pigment. **(B)** is correct.

16. D**Difficulty:** Medium

Getting to the Answer: You need to find the price of a share of the stock on October 1. You know that the price of a share was \$75 on August 1 and that on September 1 the price was \$10 higher than it was on August 1. Thus, on September 1, it was $\$75 + \$10 = \$85$.

The question also states that the September 1 price is 80% of the October 1 price. Set up an equation where p represents the October 1 price:

$$\begin{aligned} 0.8p &= \$85 \\ p &= \frac{\$85}{0.8} \\ p &= \$106.25 \end{aligned}$$

The question asks for the price to the nearest dollar, so **(D)** is correct.

17. C**Difficulty:** Easy

Getting to the Answer: The formula for percent increase or decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$. In this case, that's $\frac{1,572 - 1,494}{1,494} \times 100\% \approx 5.2\%$. Therefore, **(C)** is correct.

If you chose **(B)**, you likely divided by the new amount, \$1,572, instead of the original amount, \$1,494.

18. C**Difficulty:** Easy

Getting to the Answer: The formula for percent increase or decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$. Since the price per share of stock started at \$35 and ended up at \$49, that's $\frac{49 - 35}{35} \times 100\% = 40\%$.

Therefore, **(C)** is correct.

19. B

Difficulty: Medium

Getting to the Answer: The question asks for a percent decrease in the number of coins from the larger jar to the smaller one. The formula for percent decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$. Jar X has 75 coins and jar Y

has 54 coins. The phrase “less than” means that you’re calculating percent decrease from a starting value of 75 coins (the “original amount”); the calculation is

$$\frac{75 - 54}{75} \times 100\% = \frac{21}{75} \times 100\% = 28\%.$$

Therefore, (B) is correct.

20. D

Difficulty: Medium

Strategic Advice: Begin by calculating the number of seniors and then figure out what percent greater this number is than the number of sophomores.

Getting to the Answer: The number of seniors is 75% greater than the number of juniors, so that is $80 + (0.75 \times 80)$, or $1.75 \times 80 = 140$.

The formula for percent increase is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$. In this case, the actual change is the number of seniors minus the number of sophomores, $140 - 50$. The original amount is the number of sophomores because the question asks for a percent “greater than”—greater than the original amount, which is sophomores, or 50:

$$\frac{140 - 50}{50} \times 100\% = \frac{90}{50} \times 100\% = 180\%.$$

Thus, (D) is correct.

21. C

Difficulty: Hard

Strategic Advice: When presented with a two-part percent change scenario, you cannot simply add the two percents; you have to calculate the second percent change on the adjusted value that results from the first percent change.

Getting to the Answer: The price of the phone goes through two different changes: an initial discount of 36% and a second reduction of 25% from that discounted price. Because you don’t know y , the original price of the phone, you can pick a number to make calculations easier.

Usually, the best number to pick when calculating the percent change of an unknown value is 100, so assume that the initial price of the phone was \$100 (the numbers don’t have to be realistic, just easy to work with). Now, calculate the resulting price after the first discount: 36% of \$100 is $0.36 \times \$100 = \36 , so the new price of the phone will be $\$100 - \$36 = \$64$.

Next, calculate the change in price after an additional 25% off of the current price of \$64: 25% of \$64 is $0.25 \times \$64 = \16 , so the final price will be $\$64 - \$16 = \$48$. (Note you could have also calculated the new price by subtracting the percent discount from 100 percent: $100\% - 25\% = 75\%$, so $0.75 \times \$64 = \48 .)

The formula for percent change is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$.

Use the starting price of \$100 and the final price of \$48:

$$\frac{100 - 48}{100} \times 100\% = \frac{52}{100} \times 100\% = 52\%.$$

Thus, (C) is correct.

22. C

Difficulty: Hard

Getting to the Answer: Although the full-time and part-time employees are separate groups, the total number of employees in each group is the same. Thus, you don’t need to calculate the individual number of full- and part-time employees who have benefits. Instead, find the difference as a percent, then find that percent of 250 to get the answer in one step.

Be careful: 74 percent of part-time employees *don’t* qualify for benefits. This means that $100\% - 74\% = 26\%$ of them do qualify. Since 92% of full-time employees qualify for benefits, the percent difference is $92 - 26 = 66$. Find 66% of 250: $250 \times 0.66 = 165$. (C) is correct.

23. 59

Difficulty: Hard**Category:** Percents

Getting to the Answer: Don't switch on autopilot and do five separate cost increase calculations because you can do just one and save time. First, determine the total increase from 2014–15 through 2019–20: $\$15,564 - \$12,192 = \$3,372$. Dividing $\$3,372$ by 5 (the number of increases) gives $\$674.40$, the average increase per year. To determine what portion of this amount is fees, find 8.75% of $\$674.40$: $0.0875 \times \$674.40 = \59.01 . Rounded to the nearest dollar, the correct answer is 59. Grid in **59**.

24. C

Difficulty: Easy**Category:** Percent Change

Getting to the Answer: The formula for percent increase or decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$.

In this case, that works out to

$$\frac{27 - 20}{20} \times 100\% = \frac{7}{20} \times 100\% = 0.35 \times 100\% = 35\%.$$

Hence, **(C)** is correct.

25. D

Difficulty: Hard**Category:** Percent Change

Strategic Advice: Begin by determining how much profit company T made in 1997 so that you can calculate the percent increase in profit from 1997 to 1999.

Getting to the Answer: The question indicates that the 1998 profit of $\$375$ million was 25% greater than the 1997 profit. Therefore, you can set up this equation: $125\% \times 1997 \text{ profit} = \375 million . This can be written as $1.25p = \$375 \text{ million}$, where p represents the 1997 profit. Divide both sides by 1.25 to find that $p = \$300 \text{ million}$. Thus, the actual change in profit from 1997 to 1999 was $\$483 \text{ million} - \$300 \text{ million} = \$183 \text{ million}$.

The formula for percent increase or decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$. In this scenario, that's $\frac{183}{300} \times 100\% = 61\%$. **(D)** is correct.

26. B

Difficulty: Medium**Category:** Percent Change

Strategic Advice: Before you can calculate the percent difference between the price of a printer and the price of a calculator, you need to determine the price of the printer.

Getting to the Answer: The question indicates that the price of the printer is 16% less than the price of the radio, or 16% less than $\$75$. Therefore, the price of the printer is $(100 - 16)\% \times \$75 = 0.84 \times \$75 = \$63$. Thus, the difference in price between the calculator and the printer is $\$150 - \$63 = \$87$. With this value, you can determine what percent less this is than $\$150$.

The formula for percent increase or decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$. Hence, the price of the printer is $\frac{87}{150} \times 100\% = 58\%$ less than the price of the calculator, so **(B)** is correct.

27. C

Difficulty: Easy**Category:** Unit Conversions

Getting to the Answer: The question provides the speed of a car in miles per hour and asks for the speed in feet per second. Set up your conversion ratios so that the units cancel:

$$\begin{aligned} \frac{9 \text{ miles}}{1 \text{ hour}} &\times \frac{5,280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \\ &= 13.2 \frac{\text{feet}}{\text{second}} \end{aligned}$$

Hence, **(C)** is correct.

28. A

Difficulty: Medium

Category: Proportions

Getting to the Answer: This is a Proportions question that requires you to relate the total known number of guitars to the given ratio in order to find the unknown number of dreadnoughts, a type of guitar. First, define the relationship in the proportion with words. Then, translate from English into math. Let d equal the unknown number of dreadnoughts. Set up a proportion and cross-multiply to solve for d :

$$\begin{aligned} & \frac{\text{number of dreadnoughts in the ratio}}{\text{total number of guitars in the ratio}} \\ &= \frac{\text{actual number of dreadnoughts}}{\text{actual number of guitars}} \\ \frac{4}{4 + 15} &= \frac{d}{114} \\ 4 \times 114 &= (4 + 15)d \\ 456 &= 19d \\ 24 &= d \end{aligned}$$

Thus, (A) is correct.

Here's another way to approach this question. The total of the two values in the ratio 4:15 is 19. The total number of guitars is 114, which is 6×19 . Thus, the number of dreadnoughts is $6 \times 4 = 24$.

29. B

Difficulty: Easy

Category: Percent Change

Getting to the Answer: The formula for percent increase or decrease is $\frac{\text{actual change}}{\text{original amount}} \times 100\%$.

In this case, that works out to

$$\frac{84,600 - 74,025}{84,600} \times 100\% = \frac{10,575}{84,600} \times 100\% = 12.5\%$$

Choice (B) is correct.

30. B

Difficulty: Medium

Category: Proportions

Getting to the Answer: Call the length of each side of the pentagon p and the length of each side of the hexagon h . The perimeter of the pentagon is $5 \times p$ and the perimeter of the hexagon is $6 \times h$, so you can write

the proportion $\frac{5p}{6h} = \frac{1}{2}$. Rather than cross-multiplying, multiply both sides by $\frac{6}{5}$ which is $\frac{6}{5} \times \frac{5p}{6h} = \frac{6}{5} \times \frac{1}{2}$. This simplifies to $\frac{p}{h} = \frac{6}{10} = \frac{3}{5}$. This is another way of expressing 3:5, so (B) is correct.

31. 54

Difficulty: Hard

Category: Proportions

Getting to the Answer: Since you are given both the initial values of the different-colored candies and their values after additions and deletions in terms of ratios, you can express these as amounts relative to each other. There are the least amount of yellow candies, so you can write that the initial number of yellow candies is y , the number of white candies is $\frac{5}{4}y$, and the number of red ones is $\frac{9}{4}y$.

Now you can compare the "before and after" quantities of the red or white candies in terms of y . If you choose to look at the red candies, after the deletions and additions, the number of red candies is $\frac{4}{3}(y + 3)$ because 3 yellow candies were added. But, 7 reds were removed, so the equation is $\frac{9}{4}y - 7 = \frac{4}{3}(y + 3)$. Multiply both sides by 12 to clear the fractions: $27y - 84 = 16y + 48$. So, $11y = 132$, and $y = 12$.

The original number of white candies is $\frac{5}{4}(12) = 15$, and the original number of red is $\frac{9}{4}(12) = 27$. So, the total number of all three colors before the changes is $12 + 15 + 27 = 54$. Grid in 54.

Another way to solve this question is by picking numbers, making sure they are permissible according to the ratios. You know that the ratio of red to white is 9 to 5, and that after 7 red and 5 white are removed, the ratio will be 4 to 2—in other words, there will be twice as many red as white. Start with 9 red and 5 white candies and work up from there until you find a multiple that works:

9 red, 5 white

2 red, 0 white: doesn't work

18 red, 10 white

11 red, 5 white: doesn't work

27 red, 15 white

20 red, 10 white: this works!

The common multiplier is 3, and there are

$4 \times 3 = 12$ yellow candies. The total is

$27 + 15 + 12 = 54$ candies.

32. 50

Difficulty: Medium

Category: Unit Conversion

Getting to the Answer: Since the rate of deflation is

$100 \frac{\text{mL}}{\text{min}}$, convert the initial volume of 300 liters of air to milliliters: $300 \cancel{\text{L}} \times \frac{1,000 \text{ mL}}{1 \cancel{\text{L}}} = 300,000 \text{ mL}$.

Then, to determine the number of hours it will take for the air mattress to completely deflate, set up the calculation so that equivalent units cancel:

$$300,000 \cancel{\text{mL}} \times \frac{1 \cancel{\text{min}}}{100 \cancel{\text{mL}}} \times \frac{1 \text{ hr}}{60 \cancel{\text{min}}} = 50 \text{ hr}$$

Grid in **50**.