Answer Key

Section 20-1

1. D 2. B 3. C 4. 4.5

Section 20-2

1. B 2. C 3. D 4. C

Section 20-3

1. B 2. C 3. D 4. B

Chapter 20 Practice Test

1. D 2. A 3. B 4. D 5. C

6. C 7. 27 8. 85 9. $\frac{16}{3}$ or 5.33

Answers and Explanations

Section 20-1

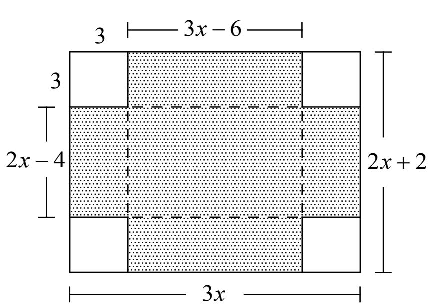
1. D

Volume of the block is $36\times20\times9-2(10\times8\times9)$, or $5{,}040\text{ cm}^3$.
Weight of the cement block = density × volume
$=1.7\text{ gram / cm}^3\times5{,}040\text{ cm}^3=8{,}568\text{ gram}$.

2. B

Volume of the aluminum block is
$10\times8\times12-8\times6\times12$, or 384 in^3.
Weight of the aluminum block = density × volume
$=0.098\text{ lb / in}^3\times384\text{ in}^3=37.632\text{ lb}$.

3. C



The dimension of the box is $(3x-6)\times(2x-4)\times3$.
Since the volume of the box is given as 162 in^3,
set up the following equation:

$(3x-6)\times(2x-4)\times3=162$
$(3x-6)\times(2x-4)=54$
$6x^2-24x+24=54$
$6x^2-24x-30=0$
$6(x^2-4x-5)=0$
$6(x-5)(x+1)=0$
$x=5$ or $x=-1$

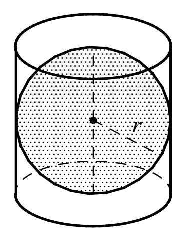
The length of the side is positive, so $x=5$.
Therefore, the dimension of the original cardboard is $(3\times5)\text{ in}\times(2\times5+2)\text{ in}$, or $15\text{ in}\times12\text{ in}$.

4. 4.5

Let h be the height of water when 2,400 cubic inches of water is added into the empty tank.
Then $20\times16\times h=2{,}400$. Solving for h yields
$h=\frac{2{,}400}{20\times16}=7.5\text{ in}$.
Since the aquarium tank is 12 inches high, the surface of water will be $12-7.5$, or 4.5 inches, from the top of the tank.

Section 20-2

1. B



Volume of the sphere $=\frac{4}{3}\pi r^3$

Volume of the cylinder $=\pi r^2\cdot\text{height}$
$=\pi r^2\cdot2r$
$=2\pi r^3$

$$\frac{\text{Volume of the sphere}}{\text{Volume of the cylinder}}=\frac{\frac{4}{3}\pi r^3}{2\pi r^3}=\frac{\frac{4}{3}}{2}=\frac{2}{3}$$

2. C

Volume of the mechanical part
$=\pi(3)^2\cdot8-\pi(1.5)^2\cdot8=54\pi\text{ in}^3$

Since $1\text{ ft}=12\text{ in}$, $1\text{ ft}^3=(12\text{ in})^3=1{,}728\text{ in}^3$.

Thus $1\text{ in}^3=\frac{1}{1{,}728}\text{ft}^3$.

Mass of the mechanical part $=$ density $\times$ volume

$= 490 \text{ lb / ft}^3 \times 54\pi \text{ in}^3 \cdot \dfrac{1 \text{ ft}^3}{1{,}728 \text{ in}^3} \approx 48.1 \text{ lb}$.

3. D

Volume of cylinder I $= \pi(r)^2 \cdot 2h = 2\pi r^2 h$

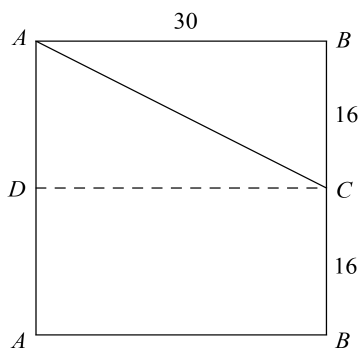
Volume of cylinder II $= \pi(2r)^2 \cdot h = 4\pi r^2 h$

$\dfrac{\text{volume of cylinder I}}{\text{volume of cylinder II}} = \dfrac{2\pi r^2 h}{4\pi r^2 h} = \dfrac{1}{2}$

Thus if the volume of cylinder I is $45\pi \text{ in}^3$, the volume of cylinder II is $90\pi \text{ in}^3$.

4. C

The figure below shows the rectangle, which was laid flat from a cut along $\overline{AB}$ on the cylinder.



$AC^2 = AB^2 + BC^2$ Pythagorean Theorem

$AC^2 = 30^2 + 16^2$ Substitution

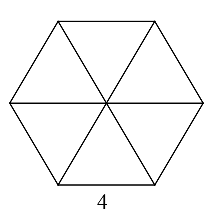
$= 900 + 256$

$= 1{,}156$

$AC = \sqrt{1{,}156} = 34$

Section 20-3

1. B



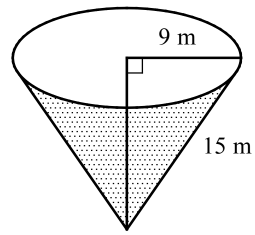
The regular hexagon consists of 6 equilateral triangles. So the area of the regular hexagon is the sum of the areas of 6 equilateral triangles. Since the area of the equilateral triangle with side length of a is $\dfrac{\sqrt{3}}{4}a^2$, the area of the equilateral triangle with side length of 4 is $\dfrac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$.

The area of the hexagon is $6 \times 4\sqrt{3} = 24\sqrt{3}$.

Volume of the pyramid

$= \dfrac{1}{3}Bh = \dfrac{1}{3}(24\sqrt{3})(7) = 56\sqrt{3}$

2. C



Let $h =$ the height of the cone.

$9^2 + h^2 = 15^2$ Pythagorean Theorem

$h^2 = 15^2 - 9^2 = 144$

$h = \sqrt{144} = 12$

Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi(9)^2(12) = 324\pi \text{ m}^3$

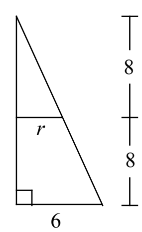
Number of minutes it takes to fill up the reservoir

$=$ total volume $\div$ rate of filling

$= 324\pi \text{ m}^3 \div 2.4 \text{ m}^3/\text{min}$

≈ 424.1 min

3. D



Let $r =$ the radius of the smaller cone.

Since the bases are parallel, the proportion $\dfrac{8}{r} = \dfrac{16}{6}$ can be used to find the radius of the smaller cone. $\dfrac{8}{r} = \dfrac{16}{6} \Rightarrow r = 3$

Volume of the remaining bottom part

$=$ volume of the cone $-$ volume of the top part

$= \dfrac{1}{3}\pi(6)^2(16) - \dfrac{1}{3}\pi(3)^2(8) = 168\pi$

4. B

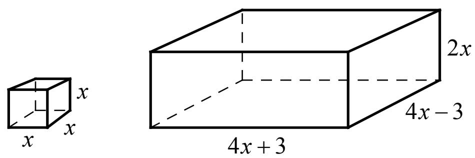
$C = 2\pi r = 10 \Rightarrow r = \dfrac{10}{2\pi} = \dfrac{5}{\pi}$

Volume of the cone

$= \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi(\dfrac{5}{\pi})^2(8) = \dfrac{200}{3\pi} \approx 21.2$

Chapter 20 Practice Test

1. D



Volume of the rectangular prism

$= (4x+3)(4x-3)(2x) = (16x^2 - 9)(2x)$

Volume of the cube $= x^3$

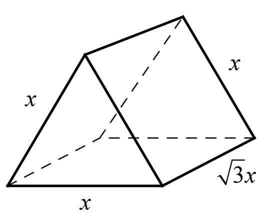
Since the volume of the rectangular prisms is 30 times the volume of the cube, the equation $(16x^2 - 9)(2x) = 30x^3$ can be used to find the value of x.

$(16x^2-9)(2x) - 30x^3 = 0$	Make one side 0.
$2x\left[(16x^2-9) - 15x^2\right] = 0$	GCF is $2x$.
$2x(x^2-9) = 0$	Simplify.
$2x(x+3)(x-3) = 0$	Factor.

$x = 0$, $x = -3$, and $x = 3$

Since the dimension has to be positive, $x = 3$ is the correct answer.

2. A



Area of the equilateral triangle with side length of x is $\dfrac{\sqrt{3}}{4}x^2$.

Volume of the triangular prism

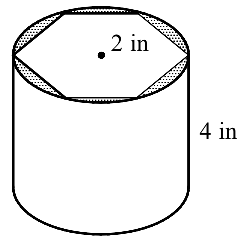
$= B \cdot h = \dfrac{\sqrt{3}}{4}x^2 \cdot \sqrt{3}x = \dfrac{3}{4}x^3$

Since the volume of the prism is given as $\dfrac{81}{4}$, the equation $\dfrac{3}{4}x^3 = \dfrac{81}{4}$ can be used to find the value of x.

$\dfrac{3}{4}x^3 = \dfrac{81}{4} \Rightarrow 3x^3 = 81 \Rightarrow x^3 = 27$

$\Rightarrow x = \sqrt[3]{27} = 3$

3. B



Area of the equilateral triangle with side length of $2 = \dfrac{\sqrt{3}}{4}(2)^2 = \sqrt{3}$.

Area of the regular hexagon $= 6\sqrt{3}$.

Volume of the hexagonal prism $= 6\sqrt{3} \cdot 4 = 24\sqrt{3}$

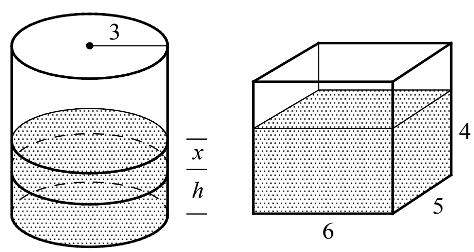
Volume of the cylinder $= \pi(2)^2 \cdot 4 = 16\pi$

The volume of the waste generated by creating the hexagonal prism from the cylinder can be found by subtracting the volume of the hexagonal prism from the volume of the cylinder.

$16\pi - 24\sqrt{3} \approx 8.69$

The volume of the waste is about 9 cubic inches.

4. D



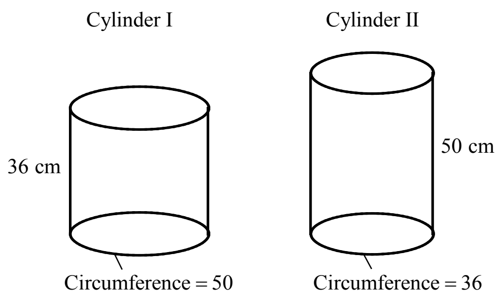
The volume of the cylinder with a radius of 3 and a height of x is $\pi(3)^2 x$, or $9\pi x$.

Volume of the water in the rectangular container is $6 \times 5 \times 4$, or 120.

To solve for x, let $9\pi x = 120$.

$x = \dfrac{120}{9\pi} \approx 4.24$

5. C



Let r_1 = the radius of cylinder I and
let r_2 = the radius of cylinder II.

$2\pi r_1 = 50 \Rightarrow r_1 = \frac{50}{2\pi} = \frac{25}{\pi}$

$2\pi r_2 = 36 \Rightarrow r_2 = \frac{36}{2\pi} = \frac{18}{\pi}$

Volume of cylinder I

$= \pi(r_1)^2 h = \pi(\frac{25}{\pi})^2(36) = \frac{22,500}{\pi}$

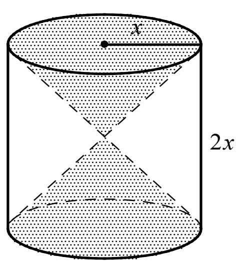
Volume of cylinder II

$= \pi(r_2)^2 h = \pi(\frac{18}{\pi})^2(50) = \frac{16,200}{\pi}$

The difference of volume between the two cylinders is $\frac{22,500}{\pi} - \frac{16,200}{\pi} \approx 2,005.3$.

Choice C is correct.

6. C



Volume of the space inside the cylinder but outside the double cone =
volume of the cylinder − volume of the two cones.

$\pi(x)^2(2x) - 2\left[\frac{1}{3}\pi(x)^2(x)\right]$

$= 2\pi x^3 - \frac{2}{3}\pi x^3 = \frac{4}{3}\pi x^3$

7. 27

Surface area of the cube $= 6s^2$

Since the surface area of the cube is given as 54 cm^2, $6s^2 = 54$.

$6s^2 = 54$ is simplified to $s^2 = 9$. Solving for s gives $s = 3$.

Volume of the cube $= s^3 = (3)^3 = 27$

8. 85

Volume of the cone $= \frac{1}{3}\pi(3)^2(10) = 30\pi$

Since the cone is 90 percent filled with shaved ice, the volume of the shaved ice is $30\pi \times 0.9$, or 27π cubic centimeters.

$27\pi \text{ cm}^3 \approx 84.8 \text{ cm}^3$

Therefore, to the nearest cubic centimeter, the volume of the shaved is 85 cm^3.

9. $\frac{16}{3}$ or 5.33

Let h = the height of the square pyramid.

Volume of the square pyramid

$= \frac{1}{3}Bh = \frac{1}{3}(6)^2 h = 12h$

Volume of the cube $= s^3 = (4)^3 = 64$

Since the square pyramid and the cube have equal volumes, $12h = 64$.

Solving for h gives $h = \frac{64}{12} = \frac{16}{3}$, or 5.33.