Answer Key

Section 18-1

1. 4	2. 6	3. 112	4.68	5. 70
6. 24	7. 240			

Section 18-2

1. 25	2. 7.5	3. 27	4. C	5. D

Section 18-3

1. 120	2. 5	3. 15	4. D	5. 108
6. 36	7. B			

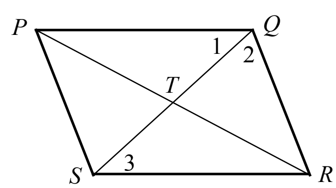
Chapter 18 Practice Test

1. C	2. B	3. 10.5	4. 174	5. D
6. 14	7. A	8. C	9. D	10. A

Answers and Explanations

Section 18-1

1. 4



$PT = \frac{1}{2}PR$ — Diagonals of ▱ bisect each other.

$x + 2y = \frac{1}{2}(32) = 16$ — Substitution

$ST = TQ$ — Diagonals of ▱ bisect each other.

$8x - y = 26$ — Substitution

$2(8x - y) = 2(26)$ — Multiply each side by 2.

$16x - 2y = 52$ — Simplify.

Add $x + 2y = 16$ and $16x - 2y = 52$.

$$\begin{array}{rl} & 16x - 2y = 52 \\ + & x + 2y = 16 \\ \hline & 17x = 68 \\ & x = 4 \end{array}$$

2. 6

Substitute 4 for x into the equation $x + 2y = 16$.

$4 + 2y = 16$

$2y = 12$

$y = 6$

3. 112

$m\angle 3 = m\angle 1$ — If $\overline{PQ} \parallel \overline{RS}$, Alternate Interior $\angle s$ are $\cong$.

$a^2 - 7 = 6a$ — Substitution

$a^2 - 6a - 7 = 0$ — Make one side 0.

$(a - 7)(a + 1) = 0$ — Factor.

$a = 7$ or $a = -1$

Discard $a = -1$, because the measure of angles in parallelogram are positive.

$m\angle 1 = 6a = 6(7) = 42$

$m\angle 2 = 10a = 10(7) = 70$

$m\angle PQR = m\angle 1 + m\angle 2$

$= 42 + 70$

$= 112$

4. 68

Since $\overline{PQ} \parallel \overline{RS}$, consecutive interior angles are supplementary. Thus, $m\angle PQR + m\angle QRS = 180$.

$112 + m\angle QRS = 180$ — $m\angle PQR = 112$

$m\angle QRS = 68$

5. 70

$m\angle QTR = m\angle PRS + m\angle 3$ — Exterior Angle Theorem

$m\angle 3 = m\angle 1 = 42$

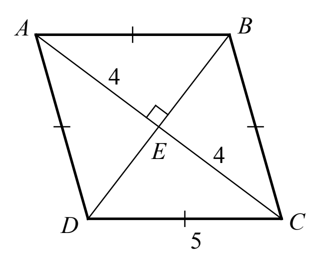
$m\angle PRS = 4a$ — Given

$= 4(7) = 28$ — $a = 7$

$m\angle QTR = 28 + 42$ — Substitution

$= 70$

6. 24



$CE^2 + DE^2 = CD^2$ — Pythagorean Theorem

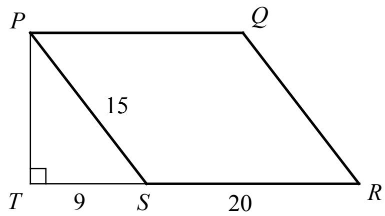
$4^2 + DE^2 = 5^2$

$DE^2 = 9$

$DE = 3$

Area of $ABCD = \frac{1}{2}AC \cdot BD = \frac{1}{2}(8)(6) = 24$

7. 240



$PT^2 + ST^2 = PS^2$ Pythagorean Theorem

$PT^2 + 9^2 = 15^2$

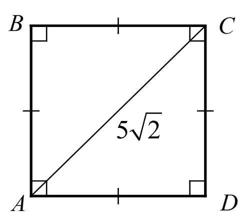
$PT^2 = 15^2 - 9^2 = 144$

$PT = \sqrt{144} = 12$

Area of $PQRS = SR \times PT = 20 \times 12 = 240$.

Section 18-2

1. 25



Let $AD = CD = s$.

$AD^2 + CD^2 = (5\sqrt{2})^2$ Pythagorean Theorem

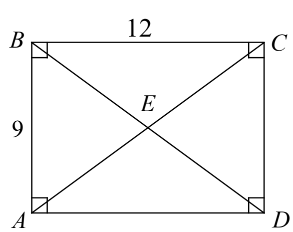
$s^2 + s^2 = 50$

$2s^2 = 50$

$s^2 = 25$

Area of $ABCD = s^2 = 25$.

2. 7.5



$AC^2 = AB^2 + BC^2$ Pythagorean Theorem

$AC^2 = 9^2 + 12^2 = 225$ Substitution

$AC = \sqrt{225} = 15$

$AE = \frac{1}{2} AC$ Diagonals of rectangle bisect each other.

$= \frac{1}{2}(15) = 7.5$

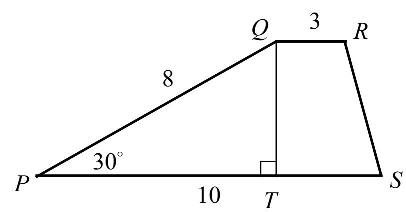
3. 27

Area of rectangle $ABCD = 12 \times 9 = 108$.
In a rectangle, diagonals divide the rectangle into four triangles of equal area. Therefore,

Area of $\Delta CED = \frac{1}{4}$ the area of rectangle $ABCD$

$= \frac{1}{4}(108) = 27$.

4. C



Draw $\overline{QT}$, which is perpendicular to $\overline{PS}$, to make triangle PQT, a 30°-60°-90° triangle.

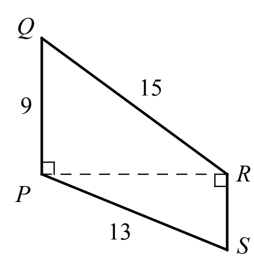
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$QT = \frac{1}{2} PQ = \frac{1}{2}(8) = 4$.

Area of trapezoid $PQRS = \frac{1}{2}(PS + QR) \cdot QT$

$= \frac{1}{2}(10 + 3) \cdot 4 = 26$

5. D



$PR^2 + PQ^2 = QR^2$ Pythagorean Theorem

$PR^2 + 9^2 = 15^2$ Substitution

$PR^2 = 15^2 - 9^2 = 144$

$PR = \sqrt{144} = 12$

$12^2 + RS^2 = 13^2$ Pythagorean Theorem

$RS^2 = 13^2 - 12^2 = 25$

$RS = \sqrt{25} = 5$

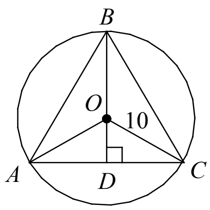
Area of trapezoid $PQRS$

$= \frac{1}{2}(PQ + RS) \cdot PR = \frac{1}{2}(9 + 5) \cdot 12$

$= 84$

Section 18-3

1. 120



$m\angle AOB = m\angle BOC = m\angle AOC = \frac{1}{3}(360) = 120$

2. 5

$m\angle COD = \frac{1}{2} m\angle AOC = \frac{1}{2}(120) = 60$

Since triangle COD is a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg.

Therefore, $OD = \frac{1}{2} CO = \frac{1}{2}(10) = 5$.

3. 15

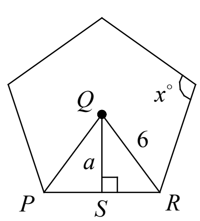
In a circle all radii are equal in measure.
Therefore, $AO = BO = CO = 10$.
$BD = BO + OD = 10 + 5 = 15$

4. D

In a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times as long as the shorter leg. Therefore,
$CD = \sqrt{3} OD = 5\sqrt{3}$
$AC = 2CD = 10\sqrt{3}$

Area of ΔABC
$= \frac{1}{2}(AC)(BD) = \frac{1}{2}(10\sqrt{3})(15) = 75\sqrt{3}$

5. 108



The measure of each interior angle of a regular n-sided polygon is $\frac{(n-2)180}{n}$. Therefore,

$x = \frac{(5-2)180}{5} = 108$.

6. 36

$m\angle PQR = \frac{360}{5} = 72$

$m\angle RQS = \frac{1}{2} m\angle PQR = \frac{1}{2}(72) = 36$

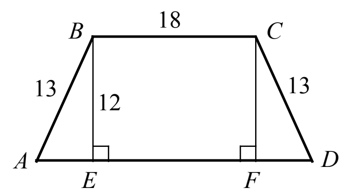
7. B

In triangle RQS, QR is the hypotenuse and QS is adjacent to $\angle RQS$. Therefore the cosine ratio can be used to find the value of a.

$\cos \angle RQS = \frac{\text{adjacent to } \angle RQS}{\text{hypotenuse}} = \frac{a}{6}$

Chapter 18 Practice Test

1. C



$AE^2 + BE^2 = AB^2$ Pythagorean Theorem
$AE^2 + 12^2 = 13^2$

$AE^2 = 13^2 - 12^2 = 25$

$AE = \sqrt{25} = 5$

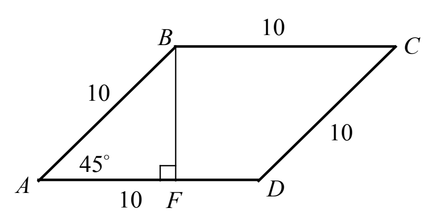
Also $DF = 5$.

$AD = AE + EF + DF = 5 + 18 + 5 = 28$

Area of trapezoid $= \frac{1}{2}(AD + BC) \cdot BF$

$= \frac{1}{2}(28 + 18) \cdot 12 = 276$

2. B



Draw $\overline{BF}$ perpendicular to $\overline{AD}$ to form a 45°-45°-90° triangle.

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg. Therefore, $\sqrt{2}BF = AB$.

$\sqrt{2}BF = 10$ Substitution

$BF = \frac{10}{\sqrt{2}} = \frac{10 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

Area of rhombus $ABCD$

$= \frac{1}{2} AD \cdot BF = \frac{1}{2}(10)(5\sqrt{2}) = 25\sqrt{2}$

3. 10.5

The length of the midsegment of a trapezoid is the average of the lengths of the bases. Therefore,

$EO = \frac{1}{2}(TP + RA)$.

$18 = \frac{1}{2}(TP + 15)$ Substitution

$2 \times 18 = 2 \times \frac{1}{2}(TP + 15)$

$36 = TP + 15$

$21 = TP$

In ΔTRP, $EZ = \frac{1}{2}TP = \frac{1}{2}(21) = 10.5$.

4. 174

Let $w =$ the width of the rectangle in meters, then $2w + 6 =$ the length of the rectangle in meters.

Area of rectangle $=$ length $\times$ width

$= (2w + 6) \times w = 2w^2 + 6w$.

Since the area of the rectangle is 1,620 square meters, you can set up the following equation.

$2w^2 + 6w = 1620$

$2w^2 + 6w - 1620 = 0$ Make one side 0.

$2(w^2 + 3w - 810) = 0$ Common factor is 2.

Use the quadratic formula to solve the equation, $w^2 + 3w - 810 = 0$.

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-810)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{3249}}{2} = \frac{-3 \pm 57}{2}$$

Since the width is positive, $w = \frac{-3 + 57}{2} = 27$.

The length is $2w + 6 = 2(27) + 6 = 60$.

The perimeter of the rectangle is

$2(\text{length} + \text{width}) = 2(60 + 27) = 174$

5. D

Area of an equilateral triangle with side length of $a = \frac{\sqrt{3}}{4}a^2$. Since the area of the equilateral triangle is given as $25\sqrt{3}$, you can set up the following equation.

$\frac{\sqrt{3}}{4}a^2 = 25\sqrt{3}$

$a^2 = 25\sqrt{3} \cdot \frac{4}{\sqrt{3}} = 100$

The area of each square is a^2, or 100, so the sum of the areas of the three squares is 3×100, or 300.

6. 14

Let $w =$ the width of the rectangle.

The perimeter of the rectangle is given as $5x$.

Perimeter of rectangle $= 2(\text{length} + \text{width})$

$5x = 2(\frac{3}{2}x + w)$

$5x = 3x + 2w$

$2x = 2w$

$x = w$

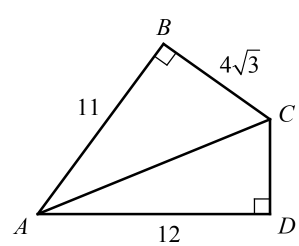
Area of rectangle $=$ length $\times$ width $= 294$

$\frac{3}{2}x \cdot x = 294$

$x^2 = 294 \cdot \frac{2}{3} = 196$

$x = \sqrt{196} = 14$

7. A



$AC^2 = AB^2 + BC^2$ Pythagorean Theorem

$AC^2 = 11^2 + (4\sqrt{3})^2$ Substitution

$AC^2 = 121 + 48 = 169$

$AC = \sqrt{169} = 13$

$AC^2 = AD^2 + CD^2$ Pythagorean Theorem

$169 = 12^2 + CD^2$ Substitution

$25 = CD^2$

$5 = CD$

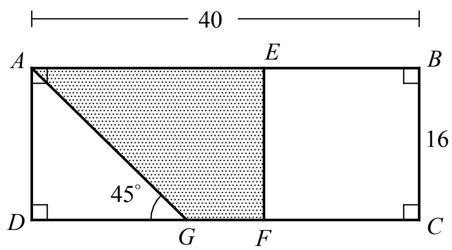
The area of region $ABCD$ is the sum of the area of ΔABC and the area of ΔADC.

Area of the region $ABCD$

$= \frac{1}{2}(11)(4\sqrt{3}) + \frac{1}{2}(12)(5)$

$= 22\sqrt{3} + 30$

8. C



Since $BCFE$ is a square,

$BC = BE = CF = EF = 16$.

$AE = AB - BE$

$= 40 - 16 = 24$

Triangle AGD is a 45°-45°-90° triangle.

In a 45°-45°-90° triangle, the length of the two legs are equal in measure. Therefore,

$AD = DG = 16$.

$FG = DC - DG - CF$

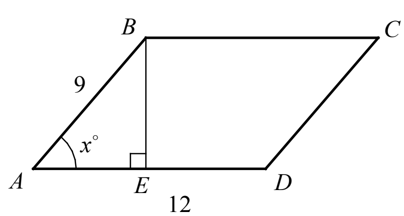
$= 40 - 16 - 16 = 8$

Area of the shaded region

$= \frac{1}{2}(AE + FG) \cdot EF$

$= \frac{1}{2}(24 + 8) \cdot 16 = 256$

9. D



Draw $\overline{BE}$ perpendicular to $\overline{AD}$.

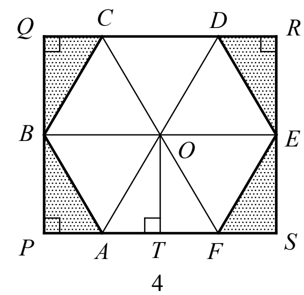
In ΔABE, $\sin x^\circ = \frac{BE}{9}$.

Therefore, $BE = 9\sin x^\circ$.

Area of parallelogram $ABCD$

$= AD \times BE = 12 \times 9 \sin x^\circ$

10. A



Draw the diagonals of a regular hexagon, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.

$BE = BO + OE = 8$ and $QR = BE = 8$

Since $ABCDEF$ is a regular hexagon, the diagonals intersect at the center of the hexagon. Let the point of intersection be O. The diagonals divide the hexagon into 6 equilateral triangles with side lengths of 4. Area of each equilateral triangle with side lengths of 4 is $\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$.

Draw $\overline{OT}$ perpendicular to $\overline{PS}$.

Triangle AOT is a 30°-60°-90° triangle.

Therefore, $AT = \frac{1}{2}AO = \frac{1}{2}(4) = 2$ and

$OT = \sqrt{3}AT = 2\sqrt{3}$.

In rectangle $PQRS$, $RS = 2OT = 2(2\sqrt{3}) = 4\sqrt{3}$.

Area of rectangle $PQRS = QR \times RS$

$= 8 \times 4\sqrt{3} = 32\sqrt{3}$.

Area of regular hexagon $ABCDEF$

$= 6 \times$ area of the equilateral triangle

$= 6 \times 4\sqrt{3} = 24\sqrt{3}$

Area of shaded region

$=$ area of rectangle $-$ area of hexagon

$= 32\sqrt{3} - 24\sqrt{3} = 8\sqrt{3}$.