CHAPTER 7
Percents

7-1. Percent of Change

The word **percent** means *hundredth* or *out of every hundred*.

To write a decimal or a fraction as a percent, multiply the decimal or the fraction by 100 and add the % sign. Convert the fraction to decimal.

To write a percent as a decimal or a fraction, multiply the percent by $\frac{1}{100}$, and drop the % sign.

Simplify the fraction.

Example 1 □ Write each decimal or fraction as a percent.

a. 0.65 b. $\frac{3}{16}$

Solution □ a. $0.65 = 0.65 \times 100\% = 65\%$

Multiply the decimal by 100 and add the % sign.

c. $\frac{3}{16} = \frac{3}{16} \times 100\% = \frac{300}{16}\% = 18.75\%$

Multiply the fraction by 100 and add the % sign. Convert the fraction to decimal.

Example 2 □ Write 175% as a decimal and a fraction.

Solution □ $175\% = 175 \times \frac{1}{100} = \frac{175}{100} = 1.75$

$175\% = 175 \times \frac{1}{100} = \frac{175}{100} = \frac{7}{4}$

Multiply the amount of percent by $\frac{1}{100}$, and drop the % sign. Simplify the fraction.

The percent a quantity increases or decreases from its original amount is the **percent of change**.

percent increase $= \frac{\text{amount of increase}}{\text{original amount}}$ **percent decrease** $= \frac{\text{amount of decrease}}{\text{original amount}}$

Example 3 □ a. A \$300 tablet is on sale for \$234. What is the percent of discount?

b. The population of Sunny Hills increased from 12,000 to 15,840 in ten years. What is the percent increase of the population?

Solution □ a. $\text{percent discount} = \frac{\text{amount of discount}}{\text{original amount}}$

$= \frac{300 - 234}{300} = \frac{66}{300} = 0.22$

There was a 22% discount.

b. $\text{percent increase} = \frac{\text{number of increase}}{\text{original number}}$

$= \frac{15,840 - 12,000}{12,000} = \frac{3,840}{12,000} = 0.32$

There was a 32% increase in population.

Exercises - Percent of Change

1

Which of the following is equivalent to 0.03 % of 4?

A) 0.12

B) 0.012

C) 0.0012

D) 0.00012

2

$\frac{1}{400} =$

A) 0.25%

B) 0.025%

C) 0.0025%

D) 0.00025%

3

The quantities x and y are positive. If x is decreased by 20 percent and y is increased by 20 percent, then the product of x and y is

A) unchanged

B) decreased by 4%

C) increased by 5%

D) decreased by 6%

4

By what percent is 4.5×10^5 greater than 9×10^4?

A) 200%

B) 400%

C) 500%

D) 600%

5

The temperature increased from 60°F to 72°F. What is the percent increase in temperature?

A) 15%

B) $\frac{50}{3}\%$

C) 20%

D) $\frac{70}{3}\%$

6

This year's enrollment in Mesa School District is 6,000, which is 20 percent higher than last year's. What was last year's enrollment in Mesa School District?

7

If 125% of x is 80 and x is n % of 400, what is the value of n?

7-2. Percents and Equations

You can solve a percent problem by writing and solving an equation or a proportion.
Three types of percent equations and corresponding verbal phrases are illustrated below.

	Verbal Phrase	Algebraic Expression	Equation or Proportion
1. Finding the Part	What is 15% of 72?	$n = 0.15 \times 72$	Write an equation.
		$\frac{15}{100} = \frac{n}{72}$	Write a proportion.
2. Finding the Percent	What percent of 20 is 6?	$\frac{n}{100} \times 20 = 6$	Write an equation.
		$\frac{n}{100} = \frac{6}{20}$	Write a proportion.
3. Finding the Whole	17 is 25% of what number?	$17 = 0.25 \times n$	Write an equation.
		$\frac{25}{100} = \frac{17}{n}$	Write a proportion.

Example 1 □ Translate each verbal phrase into an algebraic equation and a proportion. Then solve.

a. What is 0.3% of 4?
b. What percent of 30 is 5?
c. 64 is 250% of what number?

Solution □

a. $x = 0.003 \times 4 = 0.012$ Write an equation.

$\frac{x}{4} = \frac{0.3}{100}$ Write a proportion.

$100x = 0.3 \times 4$ Cross products

$x = 1.2 \div 100 = 0.012$ Divide by 100 and solve.

b. $\frac{p}{100} \times 30 = 5$ Write an equation.

$p = 5 \times \frac{100}{30} = \frac{50}{3}$ Multiply both sides by $\frac{100}{30}$ and simplify.

$\frac{p}{100} = \frac{5}{30}$ Write a proportion.

$30p = 500$ Cross products

$p = \frac{500}{30} = \frac{50}{3}$ Divide by 30 and simplify.

c. $64 = 2.5 \times n$ Write an equation. 250% = 2.5

$n = \frac{64}{2.5} = 25.6$ Divide each side by 2.5.

$\frac{250}{100} = \frac{64}{n}$ Write a proportion.

$250n = 6400$ Cross products

$n = \frac{6400}{250} = 25.6$ Divide each side by 250.

Exercises - Percents and Equations

1

28% of what number is 7?

2

3.6 is 240% of what number?

3

$\frac{1}{2}\%$ of 180 is what number?

4

$3\frac{1}{3}\%$ of what number is 2.5?

5

26.4 is 0.55% of what number?

6

What percent of 12 is 8?

A) 60%

B) $66\frac{2}{3}\%$

C) 75%

D) $130\frac{1}{3}\%$

7

54 is 120% of k.

Which of the following proportions could be used to solve the above expression?

A) $\frac{100}{120}=\frac{54}{k}$

B) $\frac{54}{100}=\frac{120}{k}$

C) $\frac{100}{54}=\frac{120}{k}$

D) $\frac{120}{100}=\frac{54}{k}$

8

If Kevin's monthly salary of \$4,500 is 72 percent of Paul's monthly salary, what is Paul's monthly salary?

A) \$3,240

B) \$5,150

C) \$5,870

D) \$6,250

7-3. Percent Word Problems

Mixture of Two Different Solutions

Example 1 □ How many milliliters of 65% acid solution must be added to 60 milliliters of a 40% acid solution in order to make a 50% acid solution?

Solution □ Let x = the amount of 65% acid solution added.

	Total amount ×	% acid =	Amount of acid
40% solution	60	40%	0.4×60
65% solution	x	65%	$0.65x$
New solution	$60 + x$	50%	$0.5(60 + x)$

Original amount of acid + added acid = new amount of acid

$0.4 \times 60 + 0.65x = 0.5(60 + x)$

$24 + 0.65x = 30 + 0.5x$

$0.15x = 6$

$x = 40$

Therefore, 40 milliliters of 65% acid solution must be added.

Interest and Investments

Example 2 □ Bob invested \$7,500 in stocks and bonds. The stocks pay 6.5% interest a year and the bonds pay 8% interest a year. His interest income is \$528 this year. How much money was invested in stocks?

Let x = the amount invested in stocks.

Then, $7500 - x$ = the amount invested in bonds.

	Amount invested ×	Rate =	Interest
Stock	x	.065	$0.065x$
Bond	$7500 - x$	.08	$0.08(7500 - x)$

Interest from stocks + interest from bonds = total interest income

$0.065x + .08(7500 - x) = 528$

$0.065x + 600 - .08x = 528$

$-0.015x = -72$

$x = 4800$

Therefore, \$4,800 was invested in stocks.

Discounts and Tax

Example 3 □ The sale price of a laptop is \$505.44 after 35% discount and 8% additional tax. What was the original price of the laptop before discount and tax?

Let x = the original price of the laptop before discount and tax.

$x - .35x = 0.65x$ The price of laptop after 35% discount.

$0.65x(1 + 0.08) = 0.702x$ The price of laptop after 8% of tax.

$0.702x = 505.44$

$x = 720$

The original price of the laptop was \$720.

Exercises - Percent Word Problems

1

There are n candies in a jar. If one candy is removed, what percent of the candies are left in terms of n?

A) $100(1-n)\%$

B) $100(\frac{1}{n}-1)\%$

C) $100(n-\frac{1}{n})\%$

D) $100(\frac{n-1}{n})\%$

2

The price of a cellphone was discounted by 25% and then discounted an additional 20%, to become \$348. What was the original price of the cellphone before it was discounted twice?

A) \$580.00

B) \$620.00

C) \$650.00

D) \$680.00

3

A chemist mixes a 40% acid solution and a 30% acid solution. How many liters of the 40% solution must be added to produce 50 liters of a solution that is 36% acid?

A) 24

B) 26

C) 30

D) 32

4

Victor invests part of his \$5,000 in a savings account that pays 4.5% annual simple interest. He invests the rest in bonds that pay 8% annual simple interest. Let s be the amount invested in savings and r be the amount invested in bonds. Victor's total income in one year from these investments is \$305.50. Which of the following systems of equations represents this relationship?

A) $\begin{cases} 0.045s + 0.08r = 5,000 \\ s + r = 305.50 \end{cases}$

B) $\begin{cases} 0.08s + 0.045r = 5,000 \\ s + r = 305.50 \end{cases}$

C) $\begin{cases} s + r = 5,000 \\ 0.045s + 0.08r = 305.50 \end{cases}$

D) $\begin{cases} s + r = 5,000 \\ 0.08s + 0.045r = 305.50 \end{cases}$

5

A sporting goods store added 50% profit cost and 8% tax to the price of a backpack, which then became \$129.60. What was the price of the backpack before adding profit and tax?

6

There are 800 students in a school and 45% of the students are male. If 30% of the male students and 25% of the female students play varsity sports, how many students play varsity sports?

Chapter 7 Practice Test

1

A chemist mixes x mL of a 34% acid solution with a 10% acid solution. If the resulting solution is 40 mL with 25% acidity, what is the value of x?

A) 18.5

B) 20

C) 22.5

D) 25

2

The price of a package of 4 pens is \$8.00. The same pens are sold at \$2.50 each. If Alex bought three packages of pens rather than buying 12 pens individually, the amount he saved on 12 pens is what percent of the amount he paid?

A) 12%

B) 20%

C) 25%

D) 30%

3

There are 600 bottles of sports drinks in a store. 25% of the bottles are orange flavored drinks. On Monday 30% of the orange flavored drinks in the store were sold and on Tuesday 20% of the remaining orange flavored drinks were sold. How many bottles of orange flavored drinks were sold in the two days?

A) 52

B) 58

C) 66

D) 75

4

A tablet with a list price of x dollars is discounted by 15% and then discounted an additional 12%. What is the final sale price of the tablet, in terms of x?

A) $0.73x$

B) $0.748x$

C) $0.75x$

D) $0.765x$

5

There is a total of n pairs of shoes in a store, all of which are either black or brown. If there are m pairs of brown shoes in the store, then in terms of m and n, what percent of the shoes in the store are black?

A) $\frac{m}{n}\%$

B) $\frac{n-m}{n}\%$

C) $(1-\frac{100m}{n})\%$

D) $100(1-\frac{m}{n})\%$

6

The numbers a, b, and c are positive and a equals $3.2bc$. If b is increased by 150% and c is decreased by 60%, then a is

A) increased by 90%

B) increased by 10%

C) unchanged

D) decreased by 10%

7

There are 10 history books in a bookcase. When the number of books increases by x percent, the new number of history books is 24. What is the value of x?

A) 58

B) 70

C) 120

D) 140

8

Number n is 25 less than 120 percent of itself. What is the value of n?

A) 125

B) 120

C) 105

D) 90

9

Of the 500 cars displayed in a certain car dealer, 7 percent are blue and 4 percent are red. The number of blue cars in the car dealer are what percent greater than the number of red cars?

A) 30%

B) 50%

C) 75%

D) 125%

10

If 300% of 0.18 is equivalent to 20% of b, then b is equivalent to what number?

11

Five people contributed $9,000 each toward the purchase of a sailboat. If they ended up paying $38,500 plus 8% sales tax for the boat, how much money should be refunded to each person?

12

A store used to sell an MP3 for $72, which is 50% more than the wholesale cost. At a special holiday sale, the price of the MP3 was 20% less than the wholesale cost. What was the special sale price of the MP3?

Answer Key

Section 7-1

1. C	2. A	3. B	4. B	5. C
6. 5000	7. 16			

Section 7-2

1. 25	2. 1.5	3. 0.9	4. 75	5. 4800
6. B	7. D	8. D		

Section 7-3

1. D	2. A	3. C	4. C	5. 80
6. 218				

Chapter 7 Practice Test

1. D	2. C	3. C	4. B	5. D
6. C	7. D	8. A	9. C	10. 2.7
11. 684	12. 38.4			

Answers and Explanations

Section 7-1

1. C

$$0.03\ \%\ \text{of}\ 4 = 0.03 \times \frac{1}{100} \times 4 = 0.0012$$

2. A

$$\frac{1}{400} = \frac{1}{400} \times 100\% = \frac{1}{4}\% = 0.25\%$$

3. B

$x - 0.2x$	x is decreased by 20 percent.
$= 0.8x$	Simplify.
$y + 0.2y$	y is increased by 20 percent.
$= 1.2y$	Simplify.

The product of decreased x and increased y is $0.8x \times 1.2y = 0.96xy$. So, the product is decreased by 4 percent.

4. B

Divide 4.5×10^5 by 9×10^4.

$$\frac{4.5 \times 10^5}{9 \times 10^4} = 5$$

So, $4.5 \times 10^5 = (9 \times 10^4) \times 5 = 9 \times 10^4 + 4(9 \times 10^4)$

$= 9 \times 10^4 + 400\%(9 \times 10^4)$.

Therefore, 4.5×10^5 is 400% greater than 9×10^4.

5. C

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$= \frac{72 - 60}{60} = \frac{12}{60} = \frac{1}{5} = 0.2 = 20\%$$

6. 5000

Let x = last year's enrollment in Mesa School District.

$$\underbrace{6000}_{\text{this year's enrollment}} = \underbrace{x + 0.2x}_{\text{20\% more than last year's enrollment}}$$

$$6000 = 1.2x$$

$$x = \frac{6000}{1.2} = 5000$$

7. 16

$1.25x = 80$	125% of x is 80.
$x = \frac{80}{1.25} = 64$	Solve for x.
$x = n\% \times 400$	x is n % of 400.
$x = n \times \frac{1}{100} \times 400$	Percent means $\frac{1}{100}$.
$x = n \times 4$	Simplify.
$64 = n \times 4$	Substitute 64 for x.
$16 = n$	Divide each side by 4.

Section 7-2

1. 25

$\frac{28}{100} \times n = 7$	28% of a number is 7.
$n = 7 \times \frac{100}{28}$	Multiply each side by $\frac{100}{28}$.
$n = 25$	Simplify.

2. 1.5

$3.6 = 2.4 \times n$	3.6 is 240% of a number.
$\frac{3.6}{2.4} = n$	Divide each side by 2.4.
$1.5 = n$	Simplify.

3. 0.9

$\frac{1}{2}\times\frac{1}{100}\times 180 = n$ $\quad \frac{1}{2}\%$ is $\frac{1}{2}\times\frac{1}{100}$.

$\frac{180}{200} = n$ Simplify.

$0.9 = n$ Simplify.

4. 75

$3\frac{1}{3}\times\frac{1}{100}\times n = 2.5$ $\quad 3\frac{1}{3}\%$ is $3\frac{1}{3}\times\frac{1}{100}$.

$\frac{10}{3}\times\frac{1}{100}\times n = 2.5$ Simplify.

$\frac{1}{30}n = 2.5$ Simplify.

$n = 2.5\times 30 = 75$ Multiply each side by 30.

5. 4800

$26.4 = 0.55\times\frac{1}{100}\times n$ $\quad 0.55\%$ is $0.55\times\frac{1}{100}$.

$26.4 = 0.0055n$ Simplify.

$\frac{26.4}{0.0055} = \frac{0.0055n}{0.0055}$ Divide each side by 0.0055.

$4800 = n$ Simplify.

6. B

$\underbrace{\frac{n}{100}}_{\text{what percent}} \underbrace{\times}_{\text{of}} 12 = 8$

$n = 8\cdot\frac{100}{12} \Rightarrow n = 66\frac{2}{3}$

8 is $66\frac{2}{3}\%$ of 12.

7. D

54 is 120% of k.

The above expression can be written as the equation $54 = 1.2\times k$. Or it can be written as the proportion $\frac{120}{100} = \frac{54}{k}$.

Choice D is correct.

8. D

Let x = Paul's monthly salary.

$\underbrace{4500}_{\text{Kevin's monthly salary}} = \underbrace{0.72}_{\text{72 percent}} \underbrace{\times}_{\text{of}} \underbrace{x}_{\text{Paul's monthly salary}}$

$4500 = 0.72x$

$x = \frac{4500}{0.72} = 6250$

Section 7-3

1. D

There are n candies in a jar and one candy is removed. So, $n-1$ candies are left in the jar.

The fraction of candies left in the jar is $\frac{n-1}{n}$.

Thus, the percent of candies left in the jar is $(\frac{n-1}{n})100\%$.

2. A

Let x = the original price of the cellphone.
The discounted price is 25% off the original price, so $x - 0.25x$, or $0.75x$, is the discounted price. After an additional discount of 20% off the first discounted price, the new price is $0.75x - 0.2(0.75x)$, or $0.6x$, which is the final price of \$348. Therefore, $0.6x = 348$.
Solving the equation for x yields $x = 580$.

3. C

Let x = the amount of 40% solution to be added.
Let $50 - x$ = the amount of 30% solution to be added.
x liters of 40 % acid + $(50 - x)$ liters of 30 % acid
= 50 liters of 36 % acid

$0.4x + 0.3(50 - x) = 0.36(50)$
$0.4x + 15 - 0.3x = 18$
$0.1x + 15 = 18$
$0.1x = 3$
$x = 30$

30 liters of 40% acid solution should be added.

4. C

If s is the amount invested in savings and r is the amount invested in bonds, $s + r$ represents the total amount invested, which is equal to \$5,000. Therefore, $s + r = 5000$.
If the amount invested in savings pays 4.5% interest and the amount invested in bonds pays 8% interest, $0.045s + 0.08r$ represents the total income from investment, which is equal to \$305.50. Therefore, $0.045s + 0.08r = 305.50$.

Choice C is correct.

5. 80

Let $x =$ the price of the backpack before adding profit and tax.
After 50% profit the price of the backpack will be $x + 0.5x$, or $1.5x$.
After 8% tax the price of the backpack will be $1.5x + .08(1.5x)$, or $1.62x$, which is equal to \$129.60. Therefore, $1.62x = 129.60$. Solving for x yields $x = 80$.
The price of the backpack before adding profit and tax was \$80.

6. 218

The number of male students $= 800 \times 0.45 = 360$.
The number of female students $= 800 - 360 = 440$.
30% of male students $= 360 \times 0.3 = 108$.
25% of female students $= 440 \times 0.25 = 110$.
The number of students who play varsity sports $= 108 + 110 = 218$

Chapter 7 Practice Test

1. D

If x mL of a 34% acid solution is added to a 10% acid solution and the resulting solution is 40 mL of a 25% solution, then the amount of the 10% acid solution should be $40 - x$ mL.

x mL of 34 % acid $+ (40 - x)$ mL of 10% acid
$= 40$ mL of 25 % acid

$0.34x + 0.1(40 - x) = 0.25(40)$
$0.34x + 4 - 0.1x = 10$
$0.24x = 6$
$x = 25$

2. C

The cost of 3 packages of pens is $3 \times \$8.00$, or \$24 and the cost of 12 pens bought individually is $12 \times \$2.50$, or \$30. The amount saved is $30 - 24$ dollars, or \$6. The percent of savings he saved on 12 pens of the amount he paid is $\frac{6}{24} \cdot 100\%$, or 25%.

3. C

The number of orange flavored drinks in the store $= 600 \times 0.25 = 150$.
The number of orange flavored drinks sold on Monday $= 150 \times 0.3 = 45$.
Remaining orange flavored drinks $= 150 - 45 = 105$.
The number of orange flavored drinks sold on Tuesday is 20% of the remaining orange flavored drinks, which is 105×0.2, or 21. Therefore, the number of bottles of orange flavored drinks sold in the two days is $45 + 21$, or 66.

4. B

After 15% discount, the price of the tablet is $x - 0.15x$, or $0.85x$. After an additional 12% discount, the price of the tablet is $0.85x - 0.12(0.85x)$, or $0.748x$.

5. D

$n =$ total number of shoes $m =$ the number of brown shoes. So the number of black shoes is $n - m$. The fraction of black shoes in the store is $\frac{n-m}{n}$, so the percent of black shoes in the store is $(\frac{n-m}{n}) \times 100\%$. This is equivalent to $(\frac{n}{n} - \frac{m}{n}) \times 100\%$, or $(1 - \frac{m}{n}) \times 100\%$.

6. C

If b is increased by 150%, it becomes $b + 1.5b$, or $2.5b$. If c is decreased by 60%, it becomes $c - 0.6c$, or $0.4c$. Multiplying these new values gives $a = 3.2(2.5b \times 0.4c) = 3.2(bc)$.

Therefore, the value is unchanged.

7. D

If 10 books are increased by x percent, then there will be $10 + 10 \times \frac{x}{100}$ books, which is equal to 24.

$10 + 10 \times \frac{x}{100} = 24$

$\Rightarrow 10 \times \frac{x}{100} = 14 \Rightarrow \frac{x}{10} = 14$
$\Rightarrow x = 140$

8. A

Number n is 25 less than 120 percent of itself.
$n = 1.2n - 25$
$-0.2n = -25$
$n = \frac{-25}{-0.2} = 125$

9. C

The number of blue cars $= 500 \times 0.07 = 35$
The number of red cars $= 500 \times 0.04 = 20$
Let 35 is n percent greater than 20.

Then $35 = 20 + 20 \cdot \frac{n}{100}$.

$35 - 20 = 20 + 20 \cdot \frac{n}{100} - 20$

$15 = \frac{1}{5} n$

$75 = n$

The number of blue cars is 75% greater than the number of red cars.

10. 2.7

300% of 0.18 is equivalent to 20% of b.

$3 \times 0.18 = 0.2b$	$300\% = 3$, $20\% = 0.2$
$0.54 = 0.2b$	Simplify.
$\frac{0.54}{0.2} = \frac{0.2}{0.2} b$	Divide each side by 0.2.
$2.7 = b$	Simplify.

11. 684

Total amount contributed by five people
$= \$9,000 \times 5 = \$45,000$.
The price of the sailboat after 8% tax
$= \$38,500 + 0.08 \times \$38,500 = \$41,580$.
The amount that should be refunded
$= \$45,000 - \$41,580 = \$3,420$.
Dividing \$3,420 by 5 yields \$684.

Thus \$684 should be refunded to each person.

12. 38.4

Let $m =$ the wholesale cost of MP3.
The selling price of \$72 is 50% more than the wholesale cost.

$72 = m + 0.5m$
$72 = 1.5m$
$48 = m$

The special holiday sale of the MP3 was 20% less than the wholesale cost. Therefore,
The special price of MP3

$= m - 0.2m$

$= 48 - 0.2 \times 48$ $\quad m = 48$

$= 38.4$

The special sale price of the MP3 was \$38.4.