

**Answer Key**

## Section 14-1

1. A    2. D    3. B    4. A    5. C

6.  $\frac{3}{2}$

## Section 14-2

1. C    2. D    3. D    4. A    5. 4

6. 9

## Section 14-3

1. B    2. C    3. C    4. D    5. A

## Section 14-4

1. C    2. B    3. D    4. C    5. B

## Chapter 14 Practice Test

1. A    2. C    3. C    4. D    5. A

6. B    7. D    8.  $\frac{3}{2}$  or 4    9. 310. 6    11.  $\frac{3}{2}$ **Answers and Explanations****Section 14-1**

1. A

$$\begin{aligned} & \frac{n^2}{n-4} + \frac{4n}{4-n} \\ &= \frac{n^2}{n-4} - \frac{4n}{n-4} \\ &= \frac{n^2 - 4n}{n-4} \\ &= \frac{n(\cancel{n-4})}{\cancel{n-4}} \\ &= n \end{aligned}$$

$n - 4 = -(4 - n)$

Add the numerators.

Factor and cancel.

2. D

$$\begin{aligned} & \frac{a}{a^2-1} - \frac{1}{a+1} \\ &= \frac{a}{(a+1)(a-1)} - \frac{1}{a+1} \end{aligned}$$

$a^2 - 1 = (a+1)(a-1)$

$$\begin{aligned} &= \frac{a}{(a+1)(a-1)} - \frac{1}{(a+1)} \cdot \frac{(a-1)}{(a-1)} \\ &= \frac{a - (a-1)}{(a+1)(a-1)} \quad \text{Add the numerators.} \\ &= \frac{1}{(a+1)(a-1)} = \frac{1}{a^2-1} \end{aligned}$$

3. B

$$\begin{aligned} & \frac{y^2-1}{1+\frac{1}{y}} \\ &= \frac{(y^2-1)y}{(1+\frac{1}{y})y} \quad \text{Multiply the numerator and denominator by } y. \\ &= \frac{\cancel{(y+1)}(y-1)y}{\cancel{y+1}} \quad \text{Distributive property} \\ &= y(y-1) \quad \text{Simplify.} \end{aligned}$$

4. A

$$\begin{aligned} & 1 - \frac{1}{x+1} \\ & 1 + \frac{1}{x^2-1} \\ &= \frac{(1 - \frac{1}{x+1})(x^2-1)}{(1 + \frac{1}{x^2-1})(x^2-1)} \quad \text{Multiply } x^2-1 \\ &= \frac{(x^2-1 - \frac{x^2-1}{x+1})}{(x^2-1+1)} \quad \text{Distributive property} \\ &= \frac{x^2-1 - (x-1)}{x^2} \quad \frac{x^2-1}{x+1} = \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} = x-1 \\ &= \frac{x^2-x}{x^2} \quad \text{Simplify.} \\ &= \frac{\cancel{x}(x-1)}{x^{\cancel{2}}} = \frac{x-1}{x} \quad \text{Factor and cancel.} \end{aligned}$$

5. C

$$\frac{x-3}{\frac{1}{x+2} - \frac{1}{2x-1}}$$

Multiply the numerator and the denominator by  $(x+2)(2x-1)$ .

$$\begin{aligned}
 &= \frac{(x-3)[(x+2)(2x-1)]}{\left(\frac{1}{x+2} - \frac{1}{2x-1}\right)[(x+2)(2x-1)]} \\
 &= \frac{(x-3)(x+2)(2x-1)}{\frac{(x+2)(2x-1)}{\cancel{x+2}} - \frac{(x+2)(2x-1)}{\cancel{2x-1}}} \\
 &= \frac{(x-3)(x+2)(2x-1)}{(2x-1) - (x+2)} \\
 &= \frac{\cancel{(x-3)}(x+2)(2x-1)}{\cancel{x-3}} \\
 &= (x+2)(2x-1)
 \end{aligned}$$

6.  $\frac{3}{2}$

$$\begin{aligned}
 &\frac{x^2 - xy}{2x} \div \frac{x - y}{3x^2} \\
 &= \frac{x^2 - xy}{2x} \times \frac{3x^2}{x - y} && \text{Rewrite as multiplication.} \\
 &= \frac{\cancel{x}(x - \cancel{y})}{2\cancel{x}} \times \frac{3\cancel{x}^2}{\cancel{x - y}} && \text{Factor and cancel.} \\
 &= \frac{3}{2}x^2
 \end{aligned}$$

So, if  $\frac{x^2 - xy}{2x} \div \frac{x - y}{3x^2} = ax^2$ , the value of  $a$  is  $\frac{3}{2}$ .

**Section 14-2**

1. C

$$\begin{aligned}
 \frac{x}{x-1} &= \frac{x-2}{x+1} \\
 x(x+1) &= (x-1)(x-2) && \text{Cross multiply.} \\
 x^2 + x &= x^2 - 3x + 2 && \text{FOIL} \\
 4x &= 2 && \text{Simplify.} \\
 x &= \frac{1}{2} && \text{Divide.}
 \end{aligned}$$

When  $x$  equals  $\frac{1}{2}$ , the denominator in the original equation does not have a value of 0.

The solution set is  $\left\{\frac{1}{2}\right\}$ .

2. D

$$\frac{x}{x-3} - 2 = \frac{4}{x-2}$$

Multiply each side by  $(x-3)(x-2)$ .

$$\begin{aligned}
 (x-3)(x-2)\left(\frac{x}{x-3} - 2\right) &= (x-3)(x-2)\left(\frac{4}{x-2}\right) \\
 x(x-2) - 2(x-3)(x-2) &= 4(x-3) \\
 x^2 - 2x - 2(x^2 - 5x + 6) &= 4x - 12 \\
 -x^2 + 8x - 12 &= 4x - 12 \\
 x^2 - 4x &= 0 \\
 x(x-4) &= 0 \\
 x = 0 &\text{ or } x = 4
 \end{aligned}$$

When  $x$  equals 0 or 4, the denominator in the original equation does not have a value of 0. The solution set is  $\{0, 4\}$ .

3. D

$$\begin{aligned}
 \frac{1}{x} - \frac{2}{x-2} &= \frac{-4}{x^2 - 2x} \\
 x^2 - 2x &= x(x-2)
 \end{aligned}$$

So the LCD is  $x(x-2)$ .

Multiply each side by  $x(x-2)$ .

$$\begin{aligned}
 x(x-2)\left(\frac{1}{x} - \frac{2}{x-2}\right) &= x(x-2)\left(\frac{-4}{x^2 - 2x}\right) \\
 (x-2) - 2x &= -4 \\
 -x - 2 &= -4 \\
 x &= 2
 \end{aligned}$$

When  $x$  equals 2, the denominator in the original equation has a value of 0. So, the equation has no solution.

4. A

$$\begin{aligned}
 \frac{3}{x^2 - 3x} + \frac{1}{3 - x} &= 2 \\
 \frac{3}{x^2 - 3x} - \frac{1}{x - 3} &= 2 && 3 - x = -(x - 3) \\
 x^2 - 3x &= x(x - 3) && \text{So the LCD is } x(x - 3).
 \end{aligned}$$

Multiply each side by  $x(x-3)$ .

$$\begin{aligned}
 x(x-3)\left(\frac{3}{x^2 - 3x} - \frac{1}{x - 3}\right) &= 2x(x-3) \\
 3 - x &= 2x^2 - 6x && \text{Distributive property} \\
 2x^2 - 5x - 3 &= 0 && \text{Make one side 0.} \\
 (2x+1)(x-3) &= 0 && \text{Factor.} \\
 x &= -\frac{1}{2} \text{ or } x = 3
 \end{aligned}$$

When  $x$  equals 3, the denominator in the original equation has a value of 0. Therefore,

3 cannot be a solution. The solution set is  $\left\{-\frac{1}{2}\right\}$ .

5. 4

If  $f(x) = \frac{1}{(x-a)^2 - 4(x-a) + 4}$  is undefined, the denominator  $(x-a)^2 - 4(x-a) + 4$  is equal to zero. If  $x = 6$ ,

$$(x-a)^2 - 4(x-a) + 4 = (6-a)^2 - 4(6-a) + 4 = 0.$$

The expression  $(6-a)^2 - 4(6-a) + 4$  is a perfect square, which can be rewritten as  $((6-a)-2)^2$ . The expression  $((6-a)-2)^2 = 0$  is equal to zero if  $(6-a)-2 = 0$ . Solving for  $a$  gives  $a = 4$ .

6. 9

The expression  $g(x) = \frac{1}{(x+3)^2 - 24(x+3) + 144}$  is undefined when the denominator of  $g(x)$  is zero.

$$(x+3)^2 - 24(x+3) + 144 = 0$$

$$((x+3)-12)^2 = 0$$

$$(x+3)-12 = 0$$

$$x = 9$$
**Section 14-3**

1. B

The equation of direct variation is  $y = kx$ , and the graph of direct variation always includes  $(0, 0)$ . Choice B is correct.

2. C

The distance it takes an automobile to stop varies directly as the square of its speed. Thus, by the definition of direct proportionality,  $d = kx^2$ , in which  $d$  is the stopping distance in feet,  $x$  is the speed of the car in miles per hour, and  $k$  is a constant.

$$d = kx^2$$

$$320 = k(40)^2 \quad d = 320, x = 40$$

$$320 = 1600k \quad \text{Simplify.}$$

$$\frac{320}{1600} = k \quad \text{Divide each side by 1600.}$$

$$d = \frac{320}{1600}x^2 \quad \text{Replace } k \text{ with } \frac{320}{1600}.$$

$$d = \frac{320}{1600}(50)^2 \quad \text{Substitute 50 for } x.$$

$$d = 500$$

3. C

$$y = \frac{k}{\sqrt{x}} \quad \text{Inverse variation equation}$$

$$12 = \frac{k}{\sqrt{16}} \quad y = 12 \text{ when } x = 16.$$

$$12 = \frac{k}{4} \text{ or } k = 48$$

$$y = \frac{48}{\sqrt{x}} \quad \text{Replace } k \text{ with } 48.$$

$$y = \frac{48}{\sqrt{100}} \quad x = 100$$

$$y = \frac{48}{10} = 4.8$$

4. D

$$L = \frac{k}{d^2}$$

$$9 = \frac{k}{2^2} \quad L = 9 \text{ and } d = 2$$

$$36 = k$$

5. A

$$\frac{L \text{ measured at distance } d}{L \text{ measured at distance } 1.5d}$$

$$= \frac{\frac{36}{d^2}}{\frac{36}{(1.5d)^2}}$$

$$= \frac{36}{d^2} \cdot \frac{(1.5d)^2}{36} = \frac{2.25d^2}{d^2}$$

$$= 2.25 = 2\frac{1}{4} = \frac{9}{4}$$

**Section 14-4**

1. C

Working together, they can finish painting the house in  $x$  days. So  $\frac{1}{x}$  is the portion of the house painting job they can finish in one day. Choice C is correct.

2. B

$$\frac{1}{4} + \frac{1}{6} = \frac{1}{x}$$

$$12x\left(\frac{1}{4} + \frac{1}{6}\right) = 12x\left(\frac{1}{x}\right) \quad \text{LCD is } 12x$$

$$3x + 2x = 12 \quad \text{Distributive property}$$

$$5x = 12 \quad \text{Simplify.}$$

$$x = \frac{12}{5} = 2\frac{2}{5} \quad \text{Divide each side by 5.}$$

3. D

Let  $a$  be the number of days it takes printer  $A$  to finish the job alone, let  $b$  be the number of days it takes printer  $B$  to finish the job alone, and let  $c$  be the number of days it takes printer  $C$  to finish the job alone. Then their respective work rates are

$\frac{1}{a}$ ,  $\frac{1}{b}$ , and  $\frac{1}{c}$ . If three printers  $A$ ,  $B$ , and  $C$ , working together at their respective constant rates, can finish a job in  $4\frac{1}{2}$  hours, you can set up

the equation  $4\frac{1}{2}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1$ . If printers,

$A$  and  $B$ , working together at their respective constant rates, can finish a job in 6 hours, you can set up the equation  $6\left(\frac{1}{a} + \frac{1}{b}\right) = 1$ .

Solving the two equations for  $\frac{1}{a} + \frac{1}{b}$  gives

$$4\frac{1}{2}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1 \Rightarrow \frac{9}{2}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{2}{9} \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{2}{9} - \frac{1}{c} \text{ and}$$

$$6\left(\frac{1}{a} + \frac{1}{b}\right) = 1 \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{1}{6}.$$

Substituting  $\frac{2}{9} - \frac{1}{c}$  for  $\frac{1}{a} + \frac{1}{b}$  gives  $\frac{2}{9} - \frac{1}{c} = \frac{1}{6}$ .

Multiply by  $18c$  on each side of the equation and simplify.

$$18c\left(\frac{2}{9} - \frac{1}{c}\right) = 18c\left(\frac{1}{6}\right)$$

$$4c - 18 = 3c$$

$$c = 18$$

4. C

Let  $b$  be the number of minutes for his brother to do the job alone. Since the part of the job Mike does in 32 minutes plus the part of the job his brother does in 32 minutes equals one whole job, you can set up the following equation.

$$32\left(\frac{1}{48} + \frac{1}{b}\right) = 1$$

$$48b \cdot 32\left(\frac{1}{48} + \frac{1}{b}\right) = 48b \cdot 1 \quad \text{LCD is } 48b.$$

$$32b + 1536 = 48b \quad \text{Simplify.}$$

$$16b = 1536$$

$$b = 96$$

5. B

If James can do a job in 8 hours, his work rate is  $\frac{1}{8}$ . If Peter can do the same job in 5 hours, his work rate is  $\frac{1}{5}$ .

Let  $x$  = the number of hours they worked together.

$$\frac{1}{8}x + \frac{1}{5}x = \frac{13}{25}$$

$$200\left(\frac{1}{8}x + \frac{1}{5}x\right) = 200 \cdot \frac{13}{25} \quad \text{LCD is } 200.$$

$$25x + 40x = 104 \quad \text{Simplify.}$$

$$65x = 104$$

$$x = \frac{104}{65} = 1.6$$

0.6 hours is  $0.6 \times 60$  minutes, or 36 minutes.

Therefore, it took 1 hour and 36 minutes for them to finish  $\frac{13}{25}$  of the job.

### Chapter 14 Practice Test

1. A

$$\frac{a}{a-b} + \frac{b}{b-a}$$

$$= \frac{a}{a-b} - \frac{b}{a-b} \quad b-a = -(a-b)$$

$$= \frac{a-b}{a-b} \quad \text{Add the numerators.}$$

$$= 1$$

2. C

$$\frac{1}{x} - \frac{1}{y}$$

$$\frac{1}{x^2} - \frac{1}{y^2}$$

Multiply  $x^2y^2$  by the numerator and the denominator.

$$\begin{aligned} & \frac{\left(\frac{1}{x} - \frac{1}{y}\right)x^2y^2}{\left(\frac{1}{x^2} - \frac{1}{y^2}\right)x^2y^2} \\ &= \frac{xy^2 - x^2y}{y^2 - x^2} && \text{Distributive property} \\ &= \frac{xy(y-x)}{(y-x)(y+x)} \\ &= \frac{xy}{y+x} \end{aligned}$$

3. C

$$\begin{aligned} \frac{(k+1)^2}{k} &= 4k \\ (k+1)^2 &= 4k^2 && \text{Multiply by } k \text{ on each side.} \\ k^2 + 2k + 1 &= 4k^2 && \text{FOIL} \\ 0 &= 3k^2 - 2k - 1 && \text{Make one side 0.} \\ 0 &= (3k+1)(k-1) && \text{Factor.} \\ k &= -\frac{1}{3} \text{ or } k = 1 \end{aligned}$$

None of the solutions make the denominator zero, thus  $\{-\frac{1}{3}, 1\}$  is the solution set.

Choice C is correct.

4. D

$$\frac{3}{x} - \frac{x}{x+2} = \frac{2}{x+2}$$

Multiply each side of the equation by  $x(x+2)$ .

$$\begin{aligned} x(x+2)\left(\frac{3}{x} - \frac{x}{x+2}\right) &= x(x+2)\left(\frac{2}{x+2}\right) \\ 3(x+2) - x^2 &= 2x && \text{Distributive property} \\ 3x + 6 - x^2 &= 2x && \text{Distributive property} \\ 0 &= x^2 - x - 6 && \text{Make one side 0.} \\ 0 &= (x+2)(x-3) && \text{Factor.} \\ x &= -2 \text{ or } x = 3 \end{aligned}$$

When  $x$  equals  $-2$ , the denominator in the original equation has a value of 0. Therefore,  $-2$  cannot be a solution.

The solution set is  $\{3\}$ .

5. A

$$\begin{aligned} \frac{x}{x+1} + \frac{4}{x-4} &= \frac{20}{x^2 - 3x - 4} \\ x^2 - 3x - 4 &= (x+1)(x-4). \text{ So the LCD is } \\ &(x+1)(x-4). \text{ Multiply each side of the equation} \\ &\text{by } (x+1)(x-4). \\ (x+1)(x-4)\left(\frac{x}{x+1} + \frac{4}{x-4}\right) & \\ = (x+1)(x-4)\left(\frac{20}{x^2 - 3x - 4}\right) & \\ x(x-4) + 4(x+1) &= 20 && \text{Distributive property} \\ x^2 - 4x + 4x + 4 &= 20 \\ x^2 &= 16 \\ x &= 4 \text{ or } x = -4 \end{aligned}$$

When  $x$  equals 4, the denominator in the original equation has a value of 0. Therefore, 4 cannot be a solution.

The solution set is  $\{-4\}$ .

6. B

$$\begin{aligned} 1 + \frac{1}{x-1} \\ 1 - \frac{1}{x+1} \\ = \frac{(x+1)(x-1)\left(1 + \frac{1}{x-1}\right)}{(x+1)(x-1)\left(1 - \frac{1}{x+1}\right)} &&& \text{Multiply by } (x+1)(x-1). \\ = \frac{(x+1)(x-1) + (x+1)}{(x+1)(x-1) - (x-1)} &&& \text{Distributive property} \\ = \frac{x^2 - 1 + x + 1}{x^2 - 1 - x + 1} &&& \text{FOIL} \\ = \frac{x^2 + x}{x^2 - x} &&& \text{Simplify.} \\ = \frac{x(x+1)}{x(x-1)} &&& \text{Factor.} \\ = \frac{x+1}{x-1} &&& \text{Cancel and simplify.} \end{aligned}$$

7. D

If working alone Gary can load the empty truck in 3 hours, his work rate is  $\frac{1}{3}$ . If working alone his brother can load the same truck in  $x$  hours, his work rate is  $\frac{1}{x}$ . If they work together for  $t$

hours to load the empty truck, the amount of work done for  $t$  hours will be  $t(\frac{1}{3} + \frac{1}{x})$ , or  $\frac{1}{3}t + \frac{1}{x}t$ .

8.  $\frac{3}{2}$  or 4

The expression  $f(x) = \frac{5}{2(x-2)^2 - 3(x-2) - 2}$  is undefined when the denominator of  $f(x)$  is zero. Therefore, if  $2(x-2)^2 - 3(x-2) - 2$  is equal to 0,  $f(x)$  is undefined.

$$2(x-2)^2 - 3(x-2) - 2 = 0$$

Let  $z = x - 2$ , then  $2z^2 - 3z - 2 = 0$ .

$$(2z+1)(z-2) = 0 \quad \text{Factor.}$$

$$2z+1=0 \text{ or } z-2=0 \quad \text{Zero Product Property}$$

$$z = -\frac{1}{2} \text{ or } z = 2$$

Now substitute  $x - 2$  for  $z$ .

$$x - 2 = -\frac{1}{2} \text{ or } x - 2 = 2$$

The values of  $x$  that make  $f$  undefined are

$$\frac{3}{2} \text{ and } 4.$$

9. 3

$$\frac{1}{2x} + \frac{3}{10x^2} = \frac{1}{5}$$

Multiply each side of the equation by  $10x^2$ .

$$10x^2\left(\frac{1}{2x} + \frac{3}{10x^2}\right) = 10x^2\left(\frac{1}{5}\right)$$

$$5x + 3 = 2x^2 \quad \text{Distributive property}$$

$$0 = 2x^2 - 5x - 3 \quad \text{Make one side 0.}$$

$$0 = (2x+1)(x-3) \quad \text{Factor.}$$

$$x = -\frac{1}{2} \text{ or } x = 3$$

Since  $x > 0$ , the only solution is 3.

10. 6

$$\frac{ab}{a-b} \div \frac{ab^2}{b-a} = -\frac{1}{6}$$

Rewrite as multiplication.

$$\frac{ab}{a-b} \times \frac{b-a}{ab^2} = -\frac{1}{6}$$

$$\frac{\cancel{ab}}{\cancel{a-b}} \times \frac{\cancel{-(a-b)}}{\cancel{ab^2}} = -\frac{1}{6} \quad b-a = -(a-b)$$

$$\frac{-1}{b} = -\frac{1}{6}$$

Therefore, the value of  $b$  is 6.

11.  $\frac{3}{2}$

$$\frac{a + \frac{1}{2}}{a - \frac{1}{2}} = 2$$

Multiply each side of the equation by  $a - \frac{1}{2}$ .

$$a + \frac{1}{2} = 2\left(a - \frac{1}{2}\right)$$

$$a + \frac{1}{2} = 2a - 1 \quad \text{Distributive property}$$

$$\frac{3}{2} = a$$