

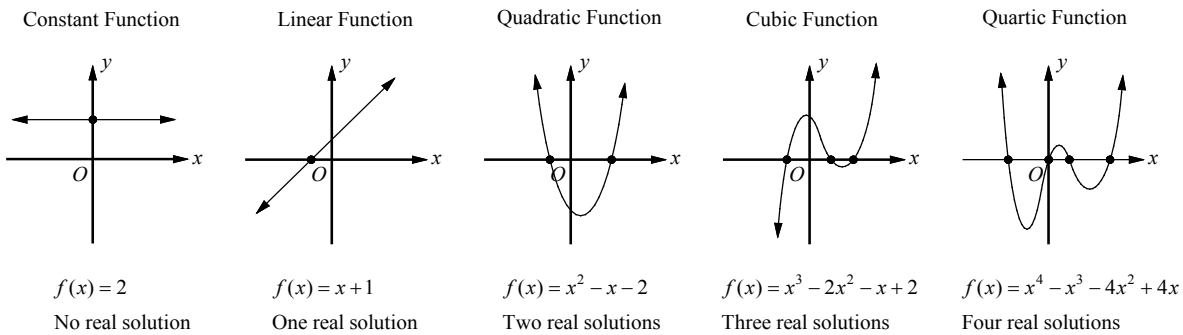
# CHAPTER 13

## Polynomial and Radical Functions

### 13-1. Polynomial Functions and Their Graphs

A **polynomial function** is a function of the form  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ , in which the coefficients  $a_n, a_{n-1}, \dots, a_2, a_1, a_0$  are real numbers and  $n$  is a nonnegative integer.

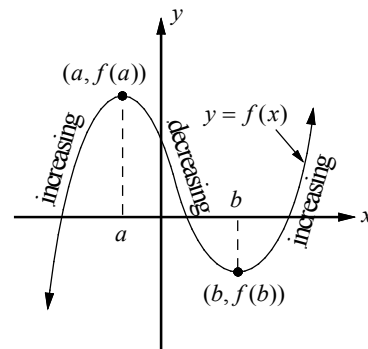
The degree of a polynomial function is its greatest exponent of  $x$ . The graphs of several polynomial functions are shown below. The maximum number of zeros, which are the  $x$ -intercepts, is equal to the degree of the function.



A function  $f$  is **increasing** on an interval if the value of  $f$  increases as  $x$  increases in the interval.

A function  $f$  is **decreasing** on an interval if the value of  $f$  decreases as  $x$  increases in the interval.

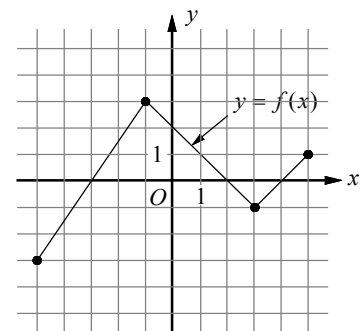
In the graph shown at the right, function  $f$  increases on the intervals  $(-\infty, a)$  and  $(b, \infty)$ , and decreases on the interval  $(a, b)$ . At a point where the graph changes from increasing to decreasing,  $f$  has a local **maximum value**, and at a point where the graph changes from decreasing to increasing,  $f$  has a local **minimum value**.



**Example 1** □ The complete graph of function  $f$  is shown at the right.

- Find the  $x$ -intercepts of  $f(x)$ .
- For what value of  $x$  is the value of  $f(x)$  at its maximum?
- Find the interval where  $f(x)$  is strictly decreasing.

- Solution** □
- The  $x$ -intercepts are  $-3$ ,  $2$ , and  $4$ .
  - The value of  $f(x)$  is maximum at  $x = -1$ .
  - $f(x)$  is strictly decreasing between  $-1$  and  $3$ .



## Exercises - Polynomial Functions and Their Graphs

1

The graph of  $f(x) = ax^3 + x^2 - 18x - 9$  intersects the  $x$ -axis at  $(3, 0)$ . What is the value of  $a$ ?

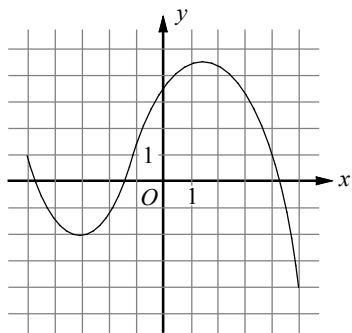
- A)  $-1$
- B)  $0$
- C)  $1$
- D)  $2$

2

In the  $xy$ -plane, the graph of function  $f$  has  $x$ -intercepts at  $-7$ ,  $-5$ , and  $5$ . Which of the following could define  $f$ ?

- A)  $f(x) = (x-7)(x^2 - 25)$
- B)  $f(x) = (x-7)(x^2 + 25)$
- C)  $f(x) = (x+7)(x^2 - 25)$
- D)  $f(x) = (x+7)(x^2 + 25)$

3

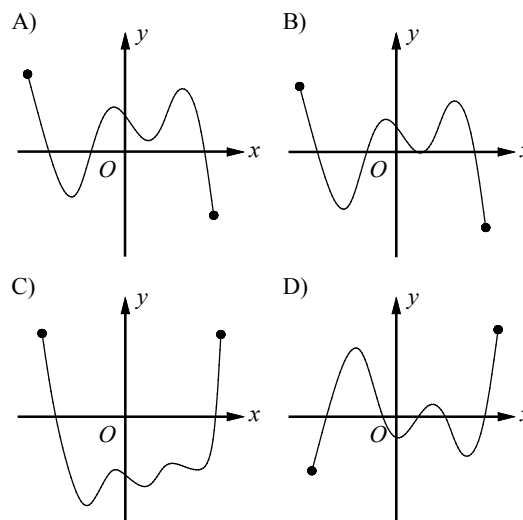


What is the minimum value of the function graphed on the  $xy$ -plane above, for  $-5 \leq x \leq 5$ ?

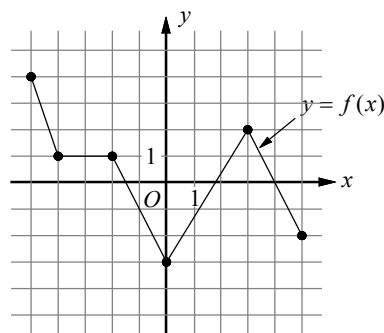
- A)  $-4$
- B)  $-3$
- C)  $-2$
- D)  $-\infty$

4

If function  $f$  has four distinct zeros, which of the following could represent the complete graph of  $f$  in the  $xy$ -plane?



5



The complete graph of function  $f$  is shown on the  $xy$ -plane above, for  $-5 \leq x \leq 5$ . Which of the following is/are true?

- I.  $f$  is strictly decreasing for  $-5 < x < 0$ .
- II.  $f(-3) = 1$
- III.  $f$  is minimum at  $x = 5$ .

- A) I only
- B) II only
- C) III only
- D) I and II only

## 13-2. Remainder Theorem and Factor Theorem

**Remainder Theorem**

If a polynomial  $f(x)$  is divided by  $x - c$ , the remainder is  $f(c)$ .

Since Dividend = Divisor  $\times$  Quotient + Remainder,

$f(x) = (x - c)q(x) + f(c)$ , in which  $q(x)$  is the quotient.

**Factor Theorem**

The polynomial  $f(x)$  has  $x - c$  as a factor if and only if  $f(c) = 0$ .

The following statements are equivalent for a polynomial  $f(x)$  and a real number  $c$ :

- $c$  is a **solution** to the equation  $f(x) = 0$ .
- $c$  is a **zero** of  $f(x)$ .
- $c$  is a **root** of  $f(x)$ .
- $x - c$  is a **factor** of  $f(x)$ .
- $f(x)$  is **divisible** by  $x - c$ .
- $c$  is an **x-intercept** of the graph of  $f(x)$ .

Example 1 □ Find the remainder of  $f(x) = x^3 + x^2 - 6x - 7$  divided by  $x + 2$ .

Solution □ To find the remainder of  $f(x)$  divided by  $x + 2 = x - (-2)$ , evaluate  $f(-2)$ .  $f(-2) = (-2)^3 + (-2)^2 - 6(-2) - 7 = 1$   
By the remainder theorem the remainder is 1.

Example 2 □ Find the value of  $a$  if  $x - 3$  is a factor of  $f(x) = x^3 - 11x + a$ .

Solution □ If  $x - 3$  is a factor of  $f(x)$ , then  $f(3) = 0$   
 $f(3) = (3)^3 - 11(3) + a = 0 \Rightarrow -6 + a = 0 \Rightarrow a = 6$

Example 3 □ Find the value of  $k$  if  $f(x) = 3(x^2 + 3x - 4) - 8(x - k)$  is divisible by  $x$ .

Solution □ If  $f(x)$  is divisible by  $x$ , since  $x = x - 0$ ,  $f(0) = 0$  by the factor theorem.  
 $f(0) = 3(0^2 + 3(0) - 4) - 8(0 - k) = -12 + 8k = 0$   
 $8k = 12 \Rightarrow k = \frac{12}{8} = \frac{3}{2}$

Example 4 □ Find the  $x$ -intercepts of  $f$  if  $f(x) = 2x^2 + x - 10$ .

<p>Solution □ <math>f(x) = 2x^2 + x - 10 = (2x + 5)(x - 2)</math> <math>(2x + 5)(x - 2) = 0</math> <math>2x + 5 = 0</math> or <math>x - 2 = 0</math> <math>x = -\frac{5}{2}</math> or <math>x = 2</math></p>	<p>Factor. Let <math>f(x) = 0</math>. Zero Product Property Solve.</p>
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The  $x$ -intercepts of  $f$  are  $-\frac{5}{2}$  and  $2$ .

## Exercises - Polynomial Functions and Their Graphs

1

If  $-1$  and  $1$  are two real roots of the polynomial function  $f(x) = ax^3 + bx^2 + cx + d$  and  $(0, 3)$  is the  $y$ -intercept of graph of  $f$ , what is the value of  $b$ ?

- A)  $-3$
- B)  $-1$
- C)  $2$
- D)  $4$

2

What is the remainder of polynomial  $p(x) = 81x^5 - 121x^3 - 36$  divided by  $x + 1$ ?

- A)  $-76$
- B)  $-36$
- C)  $4$
- D)  $6$

3

If  $x - 2$  is a factor of polynomial  $p(x) = a(x^3 - 2x) + b(x^2 - 5)$ , which of the following must be true?

- A)  $a + b = 0$
- B)  $2a - b = 0$
- C)  $2a + b = 0$
- D)  $4a - b = 0$

4

$x$	$f(x)$
$-4$	$-10$
$-3$	$0$
$-1$	$-4$
$2$	$20$

The function  $f$  is defined by a polynomial. Some values of  $x$  and  $f(x)$  are shown in the table above. Which of the following must be a factor of  $f(x)$ ?

- A)  $x + 4$
- B)  $x + 3$
- C)  $x + 1$
- D)  $x - 2$

5

$$x^3 - 8x^2 + 3x - 24 = 0$$

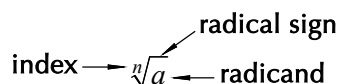
For what real value of  $x$  is the equation above true?

6

If  $x > 0$ , what is the solution to the equation  $x^4 - 8x^2 = 9$ ?

### 13-3. Radical Expressions

The symbol  $\sqrt[n]{a}$  is called a **radical**. Each part of a radical is given a name as indicated below.



#### Definition of $n$ th root

For any real numbers  $x$  or  $a$ , and any positive integer  $n$ , if  $x^n = a$ , then  $x$  is an  $n$ th root of  $a$ . If  $n$  is even,  $x = \pm\sqrt[n]{a}$ . If  $n$  is odd,  $x = \sqrt[n]{a}$ .

#### Definition of $a^{\frac{1}{2}}$ and $a^{\frac{1}{3}}$

For any nonnegative number  $a$ ,  $a^{\frac{1}{2}} = \sqrt{a}$ . For any real number  $a$ ,  $a^{\frac{1}{3}} = \sqrt[3]{a}$ .

#### Product and Quotient Property of Radicals

For any nonnegative number  $a$  or  $b$ ,  $\sqrt{ab} = \sqrt{a}\sqrt{b}$  and  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ .

For any real number  $a$  or  $b$ ,  $\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$  and  $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$  if  $b \neq 0$ .

Example 1 □ Solve.

a.  $(x-5)^4 = 16$

b.  $x^3 + 1 = -26$

Solution □ a.  $(x-5)^4 = 16$

$$x-5 = \pm\sqrt[4]{16}$$

$$x-5 = \pm 2$$

$$x = 5 \pm 2$$

$$x = 7 \text{ or } x = 3$$

Definition of  $n$ th root, for when  $n$  is even.

$$\sqrt[4]{16} = \sqrt[4]{2^4} = 2$$

Add 5 to each side.

Answer

b.  $x^3 + 1 = -26$

$$x^3 = -27$$

$$x = \sqrt[3]{-27} = (-27)^{\frac{1}{3}} = ((-3)^3)^{\frac{1}{3}}$$

$$= -3$$

Subtract 1 from each side.

Definition of  $n$ th root, for when  $n$  is odd.

Answer

Example 2 □ Simplify.

a.  $\sqrt{50}\sqrt{6}$

b.  $\sqrt{18a^2b^3}$

Solution □ a.  $\sqrt{50}\sqrt{6} = \sqrt{25}\sqrt{2}\sqrt{2}\sqrt{3}$   
 $= 5 \cdot 2 \cdot \sqrt{3} = 10\sqrt{3}$

$$\sqrt{50} = \sqrt{25}\sqrt{2}, \sqrt{6} = \sqrt{2}\sqrt{3}$$

b.  $\sqrt{18a^2b^3} = \sqrt{3^2 \cdot 2 \cdot a^2 \cdot b^2 \cdot b}$   
 $= \sqrt{3^2} \cdot \sqrt{2} \cdot \sqrt{a^2} \cdot \sqrt{b^2} \cdot \sqrt{b}$   
 $= 3 \cdot \sqrt{2} \cdot a \cdot b \cdot \sqrt{b} = 3ab\sqrt{2b}$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{3^2} = 3, \sqrt{a^2} = a, \sqrt{b^2} = b$$

A method used to eliminate radicals from a denominator is called **rationalizing the denominator**.

Binomials of the form  $\sqrt{a} + \sqrt{b}$  and  $\sqrt{a} - \sqrt{b}$  are called **conjugates**. The product of conjugates is always an integer if  $a$  and  $b$  are integers. You can use conjugates to rationalize denominators.

### Adding and Subtracting Radical Expressions

Radical expressions in which the radicands are alike can be added or subtracted in the same way that like monomials are added or subtracted.

### Multiplying Radical Expressions

Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example 3 □ Simplify.

$$\text{a. } \frac{1}{2 - \sqrt{3}}$$

$$\text{b. } (\sqrt{6} - \sqrt{2})(\sqrt{3} + 1)$$

$$\text{c. } \sqrt{50} - \sqrt{18} + \sqrt{8}$$

$$\text{d. } \sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}}$$

$$\begin{aligned} \text{Solution } \quad \square \quad \text{a. } \frac{1}{2 - \sqrt{3}} &= \frac{1}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \\ &= \frac{2 + \sqrt{3}}{2^2 - (\sqrt{3})^2} \\ &= \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3} \end{aligned}$$

The conjugate of  $2 - \sqrt{3}$  is  $2 + \sqrt{3}$ .

$$(a - b)(a + b) = a^2 - b^2$$

$$\begin{aligned} \text{b. } (\sqrt{6} - \sqrt{2})(\sqrt{3} + 1) \\ &= \sqrt{6} \cdot \sqrt{3} + \sqrt{6} \cdot 1 - \sqrt{2} \cdot \sqrt{3} - \sqrt{2} \cdot 1 \\ &= \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{3} + \sqrt{6} - \sqrt{6} - \sqrt{2} \\ &= 3\sqrt{2} - \sqrt{2} = 2\sqrt{2} \end{aligned}$$

FOIL

$$\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$$

$$\begin{aligned} \text{c. } \sqrt{50} - \sqrt{18} + \sqrt{8} \\ &= \sqrt{25 \cdot 2} - \sqrt{9 \cdot 2} + \sqrt{4 \cdot 2} \\ &= \sqrt{25}\sqrt{2} - \sqrt{9}\sqrt{2} + \sqrt{4}\sqrt{2} \\ &= 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{2} \\ &= 4\sqrt{2} \end{aligned}$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

Combine like radicals.

$$\begin{aligned} \text{d. } \sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} \\ &= \sqrt{6} - \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\ &= \sqrt{6} - \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{2} \\ &= \sqrt{6} \left( 1 - \frac{1}{3} + \frac{1}{2} \right) \\ &= \frac{7\sqrt{6}}{6} \end{aligned}$$

Rationalize the denominator.

Simplify.

Factor.

## Exercises - Radical Expressions

1

Which of the following is equal to  $a^{-\frac{1}{2}}$ ?

- A)  $-\sqrt{a}$
- B)  $\frac{1}{\sqrt{a}}$
- C)  $-\frac{1}{\sqrt{a}}$
- D)  $\frac{1}{a^2}$

2

Which of the following is equal to  $\frac{1}{3-2\sqrt{2}}$ ?

- A)  $3-\sqrt{2}$
- B)  $3+\sqrt{2}$
- C)  $3+2\sqrt{2}$
- D)  $3+4\sqrt{2}$

3

If  $(x+1)^3 = -64$ , what is the value of  $x$ ?

- A)  $-6$
- B)  $-5$
- C)  $-4$
- D)  $-3$

4

Which of the following is equal to

$$\sqrt{8} + \sqrt{18} - \sqrt{32}?$$

- A)  $\sqrt{2}$
- B)  $2\sqrt{2}$
- C)  $3\sqrt{2}$
- D)  $\sqrt{3}$

5

Which of the following is equal to

$$(1+\sqrt{3})(2-\sqrt{3})?$$

- A)  $1-\sqrt{3}$
- B)  $1+\sqrt{3}$
- C)  $-1-\sqrt{3}$
- D)  $-1+\sqrt{3}$

6

Which of the following is equal to  $b^{\frac{5}{3}}$ ?

- A)  $b \cdot \sqrt{b}$
- B)  $b \cdot \sqrt[3]{b^3}$
- C)  $b \cdot \sqrt[3]{b}$
- D)  $b \cdot \sqrt[3]{b^2}$

### 13-4. Solving Radical Equations

An equation which contains a radical with a variable in the radicand is called a **radical equation**. To solve such an equation, first isolate the radical on one side of the equation. Then square each side of the equation to eliminate the radical.

Example 1 □ Solve each equation.

a.  $\sqrt{5-2x} = 3$

b.  $4 + \sqrt{\frac{1}{2}x} = 7$

Solution □ a.  $(\sqrt{5-2x})^2 = (3)^2$

$$5 - 2x = 9$$

$$-2x = 4$$

$$x = -2$$

Square each side.

Subtract 5 from each side.

Divide each side by  $-2$ .

b.  $4 + \sqrt{\frac{1}{2}x} = 7$

Original Equation

$$\sqrt{\frac{1}{2}x} = 3$$

Subtract 3 from each side.

$$\left(\sqrt{\frac{1}{2}x}\right)^2 = (3)^2$$

Square each side.

$$\frac{1}{2}x = 9$$

Simplify.

$$x = 18$$

Multiply each side by 2.

When you square both sides of a radical equation, the resulting equation may have a solution that is not a solution of the original equation. Such a solution is called an **extraneous solution**. Therefore, you must check all the possible solutions in the original equation and disregard the extraneous solutions.

Example 2 □ Solve  $\sqrt{x+2} = x$ .

Solution □ a.  $\sqrt{x+2} = x$

Original equation

$$(\sqrt{x+2})^2 = (x)^2$$

Square each side.

$$x + 2 = x^2$$

Simplify.

$$0 = x^2 - x - 2$$

Subtract  $x$  and 2 from each side.

$$0 = (x-2)(x+1)$$

Factor.

$$x - 2 = 0 \text{ or } x + 1 = 0$$

Zero Product Property

$$x = 2 \text{ or } x = -1$$

Solve.

Check the results by substituting 2 and  $-1$  for  $x$  in the original equation.

Check:  $\sqrt{x+2} = x$

$$\sqrt{x+2} = x$$

$$\sqrt{2+2} = 2$$

$$\sqrt{-1+2} = -1$$

$$\sqrt{4} = 2$$

$$\sqrt{1} = -1$$

$$2 = 2 \quad \checkmark \text{ True}$$

$$\sqrt{1} \neq -1 \quad \times \text{ False}$$

Since  $-1$  does not satisfy the original equation, 2 is the only solution.



## Exercises - Solving Radical Equations

1

$$11 - \sqrt{2x+3} = 8$$

What is the solution set of the equation above?

- A) 0
- B) 3
- C) 6
- D) 9

2

$$\sqrt{-3x+4} = 7$$

What is the solution set of the equation above?

- A) -15
- B) -12
- C) -8
- D) -6

3

$$\sqrt{x+18} = x-2$$

What is the solution set of the equation above?

- A)  $\{-2\}$
- B)  $\{7\}$
- C)  $\{-2, 7\}$
- D)  $\{2, -7\}$

4

$$\sqrt{5x-12} = 3\sqrt{2}$$

What is the solution set of the equation above?

- A) 2
- B) 4
- C) 6
- D) 8

5

If  $a = \sqrt{3}$  and  $\sqrt{2-3x} = \frac{1}{3}a$ , what is the value of  $x$ ?

6

If  $k = 8 - \sqrt{2}$  and  $\sqrt[3]{x-k} = -2$ , what is the value of  $x^2$ ?

## 13-5. Complex Numbers

Definition of  $i$ 

$$i = \sqrt{-1} \quad \text{and} \quad i^2 = -1$$

For real numbers  $a$  and  $b$ , the expression  $a + bi$  is called a **complex number**.

Number  $a$  is called the **real part** and number  $b$  is called the **imaginary part** of the complex number  $a + bi$ .

To add or subtract complex numbers, combine the real parts and combine the imaginary parts.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

Example 1 □ Simplify.

a.  $i^{35}$

c.  $(4 - 3i) + (5 + 4i)$

b.  $\sqrt{-5} \cdot \sqrt{-10}$

d.  $2(-3 + i) - 5(1 - i)$

Solution □ a.  $i^{35} = i \cdot i^{34}$

$$= i \cdot (i^2)^{17}$$

$$= i \cdot (-1)^{17}$$

$$= -i$$

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$i^2 = -1$$

$$(-1)^{17} = -1$$

b.  $\sqrt{-5} \cdot \sqrt{-10}$

$$= (i \cdot \sqrt{5})(i \cdot \sqrt{10})$$

$$= i^2 \sqrt{50}$$

$$= (-1)(\sqrt{25} \cdot \sqrt{2})$$

$$= -5\sqrt{2}$$

$$\sqrt{-5} = \sqrt{5} \cdot \sqrt{-1} = i\sqrt{5}, \quad \sqrt{-10} = i\sqrt{10}$$

Multiply.

$$i^2 = -1, \quad \sqrt{50} = \sqrt{25} \cdot \sqrt{2}$$

Simplify.

c.  $(4 - 3i) + (5 + 4i)$

$$= (4 + 5) + (-3i + 4i)$$

$$= 9 + i$$

Combine the real parts and the imaginary parts.

Simplify.

d.  $2(-3 + i) - 5(1 - i)$

$$= -6 + 2i - 5 + 5i$$

$$= -11 + 7i$$

Multiply.

Simplify.

Example 2 □ Solve  $3x^2 + 75 = 0$ .

Solution □  $3x^2 + 75 = 0$

$$3x^2 = -75$$

$$x^2 = -25$$

$$x = \pm\sqrt{-25}$$

$$x = \pm\sqrt{25}\sqrt{-1}$$

$$x = \pm 5i$$

Subtract 75 from each side.

Divide each side by 3.

Take the square root of each side.

Product Property of Radicals

$$\sqrt{-1} = i$$

To multiply two complex numbers, use the FOIL method and use the fact that  $i^2 = -1$ .

$$(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$$

Imaginary numbers of the form  $a + bi$  and  $a - bi$  are called **complex conjugates**, and their product is the real number  $a^2 + b^2$ . This fact can be used to simplify the quotient of two imaginary numbers.

### Definition of Equal Complex Numbers

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

$$a + bi = c + di \text{ if and only if } a = c \text{ and } b = d.$$

Example 3 □ Simplify.

a.  $(6 - i)(2 + 3i)$

b.  $(\sqrt{3} + \sqrt{-2})(\sqrt{3} - \sqrt{-2})$

c.  $\frac{10}{1 + 3i}$

d.  $\frac{2 + 3i}{4 - 3i}$

Solution □ a.  $(6 - i)(2 + 3i)$

$$= 12 + 18i - 2i - 3i^2$$

$$= 12 + 16i - 3(-1)$$

$$= 15 + 16i$$

FOIL

$$i^2 = -1$$

Simplify.

b.  $(\sqrt{3} - \sqrt{-2})(\sqrt{3} + \sqrt{-2})$

$$= (\sqrt{3} + i\sqrt{2})(\sqrt{3} - i\sqrt{2})$$

$$= (\sqrt{3})^2 - (i\sqrt{2})^2$$

$$= 3 - 2i^2$$

$$= 5$$

$$\sqrt{-2} = i\sqrt{2}$$

$$(a + b)(a - b) = a^2 - b^2$$

$$i^2 = -1$$

c.  $\frac{10}{1 + 3i}$

$$= \frac{10}{1 + 3i} \cdot \frac{1 - 3i}{1 - 3i}$$

$$= \frac{10(1 - 3i)}{1 - 9i^2}$$

$$= \frac{\cancel{10}(1 - 3i)}{\cancel{10}}$$

$$= 1 - 3i$$

Rationalize the denominator.

$$(a + b)(a - b) = a^2 - b^2$$

$$i^2 = -1$$

Simplify.

d.  $\frac{2 + 3i}{4 - 3i}$

$$= \frac{2 + 3i}{4 - 3i} \cdot \frac{4 + 3i}{4 + 3i}$$

$$= \frac{8 + 6i + 12i + 9i^2}{16 - 9i^2}$$

$$= \frac{-1 + 18i}{25}$$

Rationalize the denominator.

FOIL

$$i^2 = -1$$

## Exercises - Complex Numbers

1

Which of the following is equal to  $\sqrt{-1} - \sqrt{-4} + \sqrt{-9}$ ?

- A)  $i$
- B)  $2i$
- C)  $3i$
- D)  $4i$

2

Which of the following is equal to  $\sqrt{-2} \cdot \sqrt{-8}$ ?

- A)  $-4i$
- B)  $4i$
- C)  $-4$
- D)  $4$

3

Which of the following complex numbers is equal to  $\frac{3-i}{3+i}$ ?

- A)  $\frac{9}{10} - \frac{3i}{5}$
- B)  $\frac{9}{10} + \frac{3i}{5}$
- C)  $\frac{3}{5} - \frac{3i}{5}$
- D)  $\frac{4}{5} - \frac{3i}{5}$

4

Which of the following is equal to  $\frac{1}{2}(5i-3) - \frac{1}{3}(4i+5)$ ?

- A)  $\frac{3}{2}i - \frac{5}{2}$
- B)  $\frac{7}{6}i - \frac{7}{3}$
- C)  $\frac{7}{6}i - \frac{19}{6}$
- D)  $\frac{5}{6}i - \frac{17}{6}$

5

If  $(4+i)^2 = a+bi$ , what is the value of  $a+b$ ?

6

If the expression  $\frac{3-i}{1-2i}$  is rewritten in the form  $a+bi$ , in which  $a$  and  $b$  are real numbers, what is the value of  $a+b$ ?

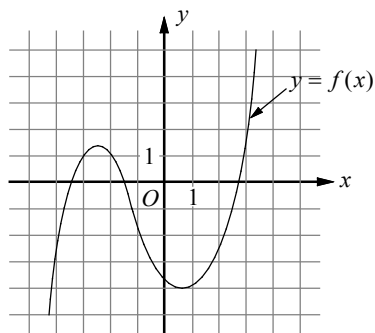
## Chapter 13 Practice Test

**1**

If the graph of  $f(x) = 2x^3 + bx^2 + 4x - 4$  intersects the  $x$ -axis at  $(\frac{1}{2}, 0)$ , and  $(-2, k)$  lies on the graph of  $f$ , what is the value of  $k$ ?

- A) -4
- B) -2
- C) 0
- D) 2

**2**



The function  $y = f(x)$  is graphed on the  $xy$ -plane above. If  $k$  is a constant such that the equation  $f(x) = k$  has one real solution, which of the following could be the value of  $k$ ?

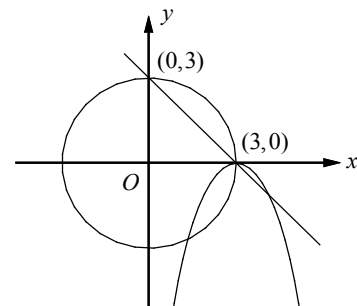
- A) -3
- B) -1
- C) 1
- D) 3

**3**

What is the value of  $a$  if  $x + 2$  is a factor of  $f(x) = -(x^3 + 3x^2) - 4(x - a)$ ?

- A) -2
- B) -1
- C) 0
- D) 1

**4**



$$\begin{aligned} x^2 + y^2 &= 9 \\ y &= -(x - 3)^2 \\ x + y &= 3 \end{aligned}$$

A system of three equations and their graphs on the  $xy$ -plane are shown above. How many solutions does the system have?

- A) 1
- B) 2
- C) 3
- D) 4

5

Which of the following complex numbers is equivalent to  $\frac{(1-i)^2}{1+i}$ ?

- A)  $-\frac{i}{2} - \frac{1}{2}$   
 B)  $-\frac{i}{2} + \frac{1}{2}$   
 C)  $-i - 1$   
 D)  $-i + 1$

6

Which of the following is equal to  $a\sqrt[3]{a}$ ?

- A)  $a^{\frac{2}{3}}$   
 B)  $a^{\frac{4}{3}}$   
 C)  $a^{\frac{5}{3}}$   
 D)  $a^{\frac{7}{3}}$

7

$$p(x) = -2x^3 + 4x^2 - 10x$$

$$q(x) = x^2 - 2x + 5$$

The polynomials  $p(x)$  and  $q(x)$  are defined above. Which of the following polynomials is divisible by  $x - 1$ ?

- A)  $f(x) = p(x) - \frac{1}{2}q(x)$   
 B)  $g(x) = -\frac{1}{2}p(x) - q(x)$   
 C)  $h(x) = -p(x) + \frac{1}{2}q(x)$   
 D)  $k(x) = \frac{1}{2}p(x) + q(x)$

8

$$\sqrt{2x+6} = x+3$$

What is the solution set of the equation above?

- A)  $\{-3\}$   
 B)  $\{-1\}$   
 C)  $\{-3, 2\}$   
 D)  $\{-3, -1\}$

9

What is the remainder when polynomial

$$p(x) = 24x^3 - 36x^2 + 14$$

is divided by  $x - \frac{1}{2}$ ?

- A) 4  
 B) 6  
 C) 8  
 D) 10

10

The function  $f$  is defined by a polynomial. If  $x + 2$ ,  $x + 1$ , and  $x - 1$  are factors of  $f$ , which of the following table could define  $f$ ?

A)

$x$	$f(x)$
-2	4
-1	0
1	0
2	0

B)

$x$	$f(x)$
-2	0
-1	4
1	0
2	0

C)

$x$	$f(x)$
-2	0
-1	0
1	4
2	0

D)

$x$	$f(x)$
-2	0
-1	0
1	0
2	4

**Answer Key**

Section 13-1

1. D    2. C    3. A    4. B    5. B

Section 13-2

1. A    2. C    3. D    4. B    5. 8

6. 3

Section 13-3

1. B    2. C    3. B    4. A    5. D

6. D

Section 13-4

1. B    2. A    3. B    4. C    5.  $\frac{5}{9}$

6. 2

Section 13-5

1. B    2. C    3. D    4. C    5. 23

6. 2

Chapter 13 Practice Test

1. C    2. D    3. B    4. A    5. C

6. B    7. B    8. D    9. C    10. D

**Answers and Explanations**

**Section 13-1**

1. D

$$f(x) = ax^3 + x^2 - 18x - 9$$

If point  $(3, 0)$  lies on the graph of  $f$ , substitute 0 for  $f$  and 3 for  $x$ .

$$0 = a(3)^3 + (3)^2 - 18(3) - 9.$$

$$0 = 27a - 54$$

$$2 = a$$

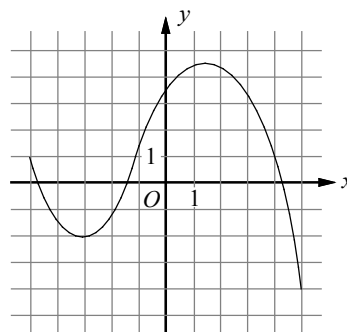
2. C

If the graph of a polynomial function  $f$  has an  $x$ -intercept at  $a$ , then  $(x - a)$  is a factor of  $f(x)$ .

Since the graph of function  $f$  has  $x$ -intercepts at  $-7$ ,  $-5$ , and  $5$ ,  $(x + 7)$ ,  $(x + 5)$ , and  $(x - 5)$  must each be a factor of  $f(x)$ . Therefore,

$$f(x) = (x + 7)(x + 5)(x - 5) = (x + 7)(x^2 - 5).$$

3. A



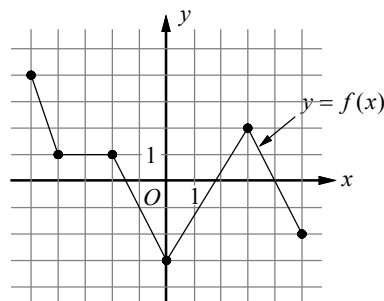
The minimum value of a graphed function is the minimum  $y$ -value of all the points on the graph. For the graph shown, when  $x = -3$ ,  $y = -2$  and when  $x = 5$ ,  $y = -4$ , so the minimum is at  $(5, -4)$  and the minimum value is  $-4$ .

4. B

A zero of a function corresponds to an  $x$ -intercept of the graph of the function on the  $xy$ -plane.

Only the graph in choice B has four  $x$ -intercepts. Therefore, it has the four distinct zeros of function  $f$ .

5. B



I.  $f$  is not strictly decreasing for  $-5 < x < 0$ , because on the interval  $-4 < x < -2$ ,  $f$  is not decreasing.

Roman numeral I is not true.

II. The coordinates  $(-3, 1)$  is on the graph of  $f$ , therefore,  $f(-3) = 1$

Roman numeral II is true.

III. For the graph shown, when  $x = 0$ ,  $y = -3$  and when  $x = 5$ ,  $y = -2$ , so  $f$  is minimum at  $x = 0$ .

Roman numeral III is not true.

## Section 13-2

1. A

If  $-1$  and  $1$  are two real roots of the polynomial function, then  $f(-1) = 0$  and  $f(1) = 0$ . Thus

$$f(-1) = a(-1)^3 + b(-1)^2 + c(-1) + d = 0 \text{ and}$$

$$f(1) = a(1)^3 + b(1)^2 + c(1) + d = 0.$$

Simplify the two equations and add them to each other.

$$\begin{array}{r} -a + b - c + d = 0 \\ + | a + b + c + d = 0 \\ \hline 2b + 2d = 0 \text{ or } b + d = 0. \end{array}$$

Also  $f(0) = 3$ , since the graph of the polynomial passes through  $(0, 3)$ .

$$f(0) = a(0)^3 + b(0)^2 + c(0) + d = 3 \text{ implies } d = 3.$$

Substituting  $d = 3$  in the equation  $b + d = 0$  gives  $b + 3 = 0$ , or  $b = -3$ .

2. C

If polynomial  $p(x) = 81x^5 - 121x^3 - 36$  is divided by  $x + 1$ , the remainder is  $p(-1)$ .

$$p(-1) = 81(-1)^5 - 121(-1)^3 - 36 = 4$$

The remainder is 4.

3. D

If  $x - 2$  is a factor for polynomial  $p(x)$ , then

$$p(2) = 0.$$

$$p(x) = a(x^3 - 2x) + b(x^2 - 5)$$

$$p(2) = a(2^3 - 2(2)) + b(2^2 - 5)$$

$$= a(8 - 4) + b(4 - 5)$$

$$= 4a - b = 0$$

4. B

If  $(x - a)$  is a factor of  $f(x)$ , then  $f(a)$  must be equal to 0. Based on the table,  $f(-3) = 0$ .

Therefore,  $x + 3$  must be a factor of  $f(x)$ .

5. 8

$$x^3 - 8x^2 + 3x - 24 = 0$$

$$(x^3 - 8x^2) + (3x - 24) = 0 \quad \text{Group terms.}$$

$$x^2(x - 8) + 3(x - 8) = 0 \quad \text{Factor out the GCF.}$$

$$(x^2 + 3)(x - 8) = 0 \quad \text{Distributive Property}$$

$$x^2 + 3 = 0 \text{ or } x - 8 = 0 \quad \text{Solutions}$$

Since  $x^2 + 3 = 0$  does not have a real solution,  $x - 8 = 0$ , or  $x = 8$ , is the only solution that makes the equation true.

6. 3

$$x^4 - 8x^2 = 9$$

$$x^4 - 8x^2 - 9 = 0 \quad \text{Make one side 0.}$$

$$(x^2 - 9)(x^2 + 1) = 0 \quad \text{Factor.}$$

$$(x + 3)(x - 3)(x^2 + 1) = 0 \quad \text{Factor.}$$

Since  $x^2 + 1 = 0$  does not have a real solution, the solutions for  $x$  are  $x = -3$  and  $x = 3$ .

Since it is given that  $x > 0$ ,  $x = 3$  is the only solution to the equation.

## Section 13-3

1. B

$$a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}}$$

2. C

$$\frac{1}{3 - 2\sqrt{2}}$$

$$= \frac{1}{3 - 2\sqrt{2}} \cdot \frac{3 + 2\sqrt{2}}{3 + 2\sqrt{2}} \quad \text{Multiply the conjugate of the denominator.}$$

$$= \frac{3 + 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{3 + 2\sqrt{2}}{9 - 8} \quad \text{Simplify.}$$

$$= 3 + 2\sqrt{2}$$

3. B

$$(x + 1)^3 = -64$$

$$x + 1 = \sqrt[3]{-64} \quad \text{Definition of cube root.}$$

$$x + 1 = -4 \quad \sqrt[3]{-64} = (-64)^{\frac{1}{3}} = -4$$

$$x = -5 \quad \text{Subtract 1 from each side.}$$

4. A

$$\begin{aligned} & \sqrt{8} + \sqrt{18} - \sqrt{32} \\ &= \sqrt{4}\sqrt{2} + \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{2} \\ &= 2\sqrt{2} + 3\sqrt{2} - 4\sqrt{2} \\ &= \sqrt{2} \end{aligned}$$



5. D

$$\begin{aligned} (1+\sqrt{3})(2-\sqrt{3}) & \\ = 2-\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3} & \text{ FOIL} \\ = 2+\sqrt{3}-3 & \text{ Combine like radicals.} \\ = -1+\sqrt{3} & \text{ Simplify.} \end{aligned}$$

6. D

$$b^{\frac{5}{3}} = b^1 \cdot b^{\frac{2}{3}} = b \cdot (b^2)^{\frac{1}{3}} = b \cdot \sqrt[3]{b^2}$$

**Section 13-4**

1. B

$$\begin{aligned} 11-\sqrt{2x+3} &= 8 \\ 11-\sqrt{2x+3}-11 &= 8-11 \quad \text{Subtract 11 from each side.} \\ -\sqrt{2x+3} &= -3 \quad \text{Simplify.} \\ (-\sqrt{2x+3})^2 &= (-3)^2 \quad \text{Square each side.} \\ 2x+3 &= 9 \quad \text{Simplify.} \\ 2x &= 6 \quad \text{Subtract 3 from each side.} \\ x &= 3 \quad \text{Divide each side by 2.} \end{aligned}$$

2. A

$$\begin{aligned} \sqrt{-3x+4} &= 7 \\ (\sqrt{-3x+4})^2 &= (7)^2 \quad \text{Square each side.} \\ -3x+4 &= 49 \quad \text{Simplify.} \\ -3x &= 45 \quad \text{Subtract 4 from each side.} \\ x &= -15 \quad \text{Divide each side by } -3. \end{aligned}$$

3. B

$$\begin{aligned} \sqrt{x+18} &= x-2 \\ (\sqrt{x+18})^2 &= (x-2)^2 \quad \text{Square each side.} \\ x+18 &= x^2-4x+4 \quad \text{Simplify.} \\ 0 &= x^2-5x-14 \quad \text{Make one side 0.} \\ 0 &= (x-7)(x+2) \quad \text{Factor.} \\ 0 &= x-7 \quad \text{or} \quad 0 = x+2 \quad \text{Zero Product Property} \\ 7 &= x \quad \text{or} \quad -2 = x \end{aligned}$$

Check each  $x$ -value in the original equation.

$$\begin{aligned} \sqrt{7+18} &= 7-2 & x &= 7 \\ \sqrt{25} &= 5 & \text{Simplify.} \\ 5 &= 5 & \text{True} \\ \sqrt{-2+18} &= -2-2 & x &= -2 \\ \sqrt{16} &= -4 & \text{Simplify.} \\ 4 &= -4 & \text{False} \end{aligned}$$

Thus, 7 is the only solution.

4. C

$$\begin{aligned} \sqrt{5x-12} &= 3\sqrt{2} \\ (\sqrt{5x-12})^2 &= (3\sqrt{2})^2 \quad \text{Square each side.} \\ 5x-12 &= 18 \quad \text{Simplify.} \\ 5x &= 30 \quad \text{Add 12 to each side.} \\ x &= 6 \quad \text{Divide by 5 on each side.} \end{aligned}$$

5.  $\frac{5}{9}$

$$\begin{aligned} \sqrt{2-3x} &= \frac{1}{3}a \\ \sqrt{2-3x} &= \frac{1}{3}\sqrt{3} & a &= \sqrt{3} \\ (\sqrt{2-3x})^2 &= \left(\frac{1}{3}\sqrt{3}\right)^2 \quad \text{Square each side.} \\ 2-3x &= \frac{1}{3} \quad \text{Simplify.} \\ -3x &= -\frac{5}{3} \quad \text{Subtract 2 from each side.} \\ -\frac{1}{3}(-3x) &= -\frac{1}{3}\left(-\frac{5}{3}\right) \quad \text{Multiply each side by } -\frac{1}{3}. \\ x &= \frac{5}{9} \quad \text{Simplify.} \end{aligned}$$

6. 2

$$\begin{aligned} \sqrt[3]{x-k} &= -2 \\ (\sqrt[3]{x-k})^3 &= (-2)^3 \quad \text{Cube each side.} \\ x-k &= -8 \quad \text{Simplify.} \\ x-(8-\sqrt{2}) &= -8 & k &= 8-\sqrt{2} \\ x-8+\sqrt{2} &= -8 \quad \text{Simplify.} \\ x+\sqrt{2} &= 0 \quad \text{Add 8 to each side.} \\ x &= -\sqrt{2} \quad \text{Subtract } \sqrt{2}. \\ (x)^2 &= (-\sqrt{2})^2 \quad \text{Square each side.} \\ x^2 &= 2 \quad \text{Simplify.} \end{aligned}$$

**Section 13-5**

1. B

$$\begin{aligned} \sqrt{-1}-\sqrt{-4}+\sqrt{-9} & \\ = i-i\sqrt{4}+i\sqrt{9} & \quad i = \sqrt{-1} \\ = i-2i+3i & \\ = 2i & \end{aligned}$$

2. C

$$\begin{aligned} & \sqrt{-2} \cdot \sqrt{-8} \\ &= i\sqrt{2} \cdot i\sqrt{8} & \sqrt{-2} = i\sqrt{2}, \sqrt{-8} = i\sqrt{8} \\ &= i^2\sqrt{16} \\ &= -4 & i^2 = -1 \end{aligned}$$

3. D

$$\begin{aligned} & \frac{3-i}{3+i} \\ &= \frac{3-i}{3+i} \cdot \frac{3-i}{3-i} & \text{Rationalize the denominator.} \\ &= \frac{9-6i+i^2}{9-i^2} & \text{FOIL} \\ &= \frac{9-6i-1}{9+1} & i^2 = -1 \\ &= \frac{8-6i}{10} & \text{Simplify.} \\ &= \frac{4-3i}{5} \text{ or } \frac{4}{5} - \frac{3i}{5} \end{aligned}$$

4. C

$$\begin{aligned} & \frac{1}{2}(5i-3) - \frac{1}{3}(4i+5) \\ &= \frac{5}{2}i - \frac{3}{2} - \frac{4i}{3} - \frac{5}{3} & \text{Distributive Property} \\ &= \frac{15}{6}i - \frac{9}{6} - \frac{8i}{6} - \frac{10}{6} & 6 \text{ is the GCD.} \\ &= \frac{7}{6}i - \frac{19}{6} & \text{Simplify.} \end{aligned}$$

5. 23

$$\begin{aligned} (4+i)^2 &= a+bi \\ 16+8i+i^2 &= a+bi & \text{FOIL} \\ 16+8i-1 &= a+bi & i^2 = -1 \\ 15+8i &= a+bi & \text{Simplify.} \\ 15 = a \text{ and } 8 = b & & \text{Definition of Equal Complex Numbers} \end{aligned}$$

Therefore,  $a+b=15+8=23$ .

6. 2

$$\begin{aligned} \frac{3-i}{1-2i} &= \frac{3-i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{3+6i-i-2i^2}{1-4i^2} \\ &= \frac{3+6i-i+2}{1+4} = \frac{5+5i}{5} = 1+i = a+bi \end{aligned}$$

Therefore,  $a=1$  and  $b=1$ , and  $a+b=1+1=2$ .**Chapter 13 Practice Test**

1. C

$$f(x) = 2x^3 + bx^2 + 4x - 4$$

$f\left(\frac{1}{2}\right) = 0$  because the graph of  $f$  intersects the  $x$ -axis at  $\left(\frac{1}{2}, 0\right)$ .

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 4 = 0$$

Solving the equation for  $b$  gives  $b=7$ .

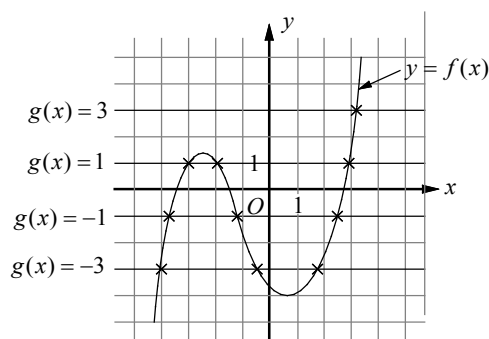
$$\text{Thus } f(x) = 2x^3 + 7x^2 + 4x - 4.$$

Also  $k = f(-2)$ , because  $(-2, k)$  lies on the graph of  $f$ .

$$k = f(-2) = 2(-2)^3 + 7(-2)^2 + 4(-2) - 4$$

Solving the equation for  $k$  gives  $k=0$ .

2. D



$g(x) = -3$  has 3 points of intersection with  $y = f(x)$ , so there are 3 real solutions.

$g(x) = -1$  has 3 points of intersection with  $y = f(x)$ , so there are 3 real solutions.

$g(x) = 1$  has 3 points of intersection with  $y = f(x)$ , so there are 3 real solutions.

$g(x) = 3$  has 1 point of intersection with  $y = f(x)$ , so there is 1 real solution.

Choice D is correct

3. B

If  $x+2$  is a factor of

$$f(x) = -(x^3 + 3x^2) - 4(x-a), \text{ then } f(-2) = 0.$$

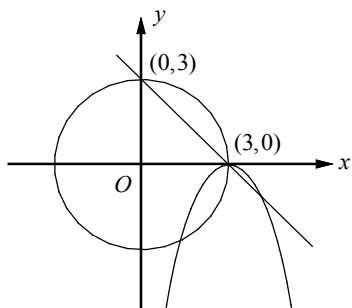
$$f(-2) = -((-2)^3 + 3(-2)^2) - 4(-2-a) = 0$$

$$-(-8+12)+8+4a=0$$

$$4+4a=0$$

$$a=-1$$

4. A



The solutions to the system of equations are the points where the circle, parabola, and line all intersect. That point is  $(3, 0)$  and is therefore the only solution to the system.

5. C

$$\begin{aligned} & \frac{(1-i)^2}{1+i} \\ &= \frac{1-2i+i^2}{1+i} && \text{FOIL the numerator.} \\ &= \frac{1-2i-1}{1+i} && i^2 = -1 \\ &= \frac{-2i}{1+i} && \text{Simplify.} \\ &= \frac{-2i}{1+i} \cdot \frac{1-i}{1-i} && \text{Rationalize the denominator.} \\ &= \frac{-2i+2i^2}{1-i^2} && \text{FOIL} \\ &= \frac{-2i-2}{2} && i^2 = -1 \\ &= -i-1 \end{aligned}$$

6. B

$$a \sqrt[3]{a} = a \cdot a^{\frac{1}{3}} = a^{1+\frac{1}{3}} = a^{\frac{4}{3}}$$

7. B

$$p(x) = -2x^3 + 4x^2 - 10x$$

$$q(x) = x^2 - 2x + 5$$

In  $p(x)$ , factoring out the GCF,  $-2x$ , yields

$$p(x) = -2x(x^2 - 2x + 5) = -2x \cdot q(x).$$

Let's check each answer choice.

$$\begin{aligned} \text{A) } f(x) &= p(x) - \frac{1}{2}q(x) \\ &= -2x \cdot q(x) - \frac{1}{2}q(x) = \left(-2x - \frac{1}{2}\right)q(x) \end{aligned}$$

$q(x)$  is not a factor of  $x-1$  and  $\left(-2x - \frac{1}{2}\right)$  is not a factor of  $x-1$ .  $f(x)$  is not divisible by  $x-1$ .

$$\begin{aligned} \text{B) } g(x) &= -\frac{1}{2}p(x) - q(x) \\ &= -\frac{1}{2}[-2x \cdot q(x)] - q(x) = (x-1)q(x) \end{aligned}$$

Since  $g(x)$  is  $x-1$  times  $q(x)$ ,  $g(x)$  is divisible by  $x-1$ .

Choices C and D are incorrect because  $x-1$  is not a factor of the polynomials  $h(x)$  and  $k(x)$ .

8. D

$$\begin{aligned} \sqrt{2x+6} &= x+3 \\ (\sqrt{2x+6})^2 &= (x+3)^2 && \text{Square each side.} \\ 2x+6 &= x^2+6x+9 && \text{Simplify.} \\ x^2+4x+3 &= 0 && \text{Make one side 0.} \\ (x+1)(x+3) &= 0 && \text{Factor.} \\ x+1=0 \text{ or } x+3=0 &&& \text{Zero Product Property} \\ x=-1 \text{ or } x=-3 \end{aligned}$$

Check each  $x$ -value in the original equation.

$$\begin{aligned} \sqrt{2(-1)+6} &= -1+3 && x=-1 \\ \sqrt{4} &= 2 && \text{Simplify.} \\ 2 &= 2 && \text{True} \\ \sqrt{2(-3)+6} &= -3+3 && x=-3 \\ 0 &= 0 && \text{True} \end{aligned}$$

Thus,  $-1$  and  $-3$  are both solutions to the equation.

9. C

Use the remainder theorem.

$$p\left(\frac{1}{2}\right) = 24\left(\frac{1}{2}\right)^3 - 36\left(\frac{1}{2}\right)^2 + 14 = 8$$

Therefore, the remainder of polynomial

$$p(x) = 24x^3 - 36x^2 + 14 \text{ divided by } x - \frac{1}{2} \text{ is } 8.$$

10. D

If  $(x-a)$  is a factor of  $f(x)$ , then  $f(a)$  must equal to 0. Thus, if  $x+2$ ,  $x+1$  and  $x-1$  are factors of  $f$ , we have  $f(-2) = f(-1) = f(1) = 0$ .

Choice D is correct.