Answer Key

Section 13-1

1. D	2. C	3. A	4. B	5. B

Section 13-2

1. A	2. C	3. D	4. B	5. 8
6. 3				

Section 13-3

1. B	2. C	3. B	4. A	5. D
6. D				

Section 13-4

1. B	2. A	3. B	4. C	5. $\frac{5}{9}$
6. 2				

Section 13-5

1. B	2. C	3. D	4. C	5. 23
6. 2				

Chapter 13 Practice Test

1. C	2. D	3. B	4. A	5. C
6. B	7. B	8. D	9. C	10. D

Answers and Explanations

Section 13-1

1. D

$f(x) = ax^3 + x^2 - 18x - 9$

If point $(3,0)$ lies on the graph of f, substitute 0 for f and 3 for x.

$0 = a(3)^3 + (3)^2 - 18(3) - 9$.

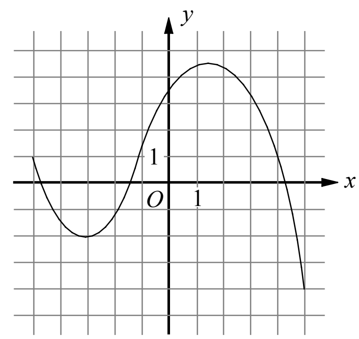
$0 = 27a - 54$

$2 = a$

2. C

If the graph of a polynomial function f has an x-intercept at a, then $(x-a)$ is a factor of $f(x)$. Since the graph of function f has x-intercepts at -7, -5, and 5, $(x+7)$, $(x+5)$, and $(x-5)$ must each be a factor of $f(x)$. Therefore, $f(x) = (x+7)(x+5)(x-5) = (x+7)(x^2-5)$.

3. A

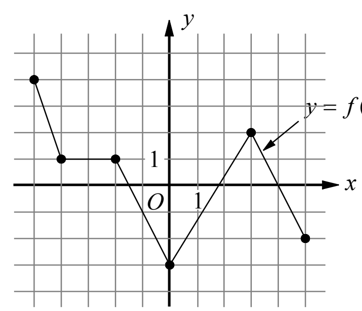


The minimum value of a graphed function is the minimum y-value of all the points on the graph. For the graph shown, when $x = -3$, $y = -2$ and when $x = 5$, $y = -4$, so the minimum is at $(5,-4)$ and the minimum value is -4.

4. B

A zero of a function corresponds to an x-intercept of the graph of the function on the xy-plane. Only the graph in choice B has four x-intercepts. Therefore, it has the four distinct zeros of function f.

5. B



I. f is not strictly decreasing for $-5 < x < 0$, because on the interval $-4 < x < -2$, f is not decreasing.

Roman numeral I is not true.

II. The coordinates $(-3,1)$ is on the graph of f, therefore, $f(-3) = 1$

Roman numeral II is true.

III. For the graph shown, when $x = 0$, $y = -3$ and when $x = 5$, $y = -2$, so f is minimum at $x = 0$.

Roman numeral III is not true.

Section 13-2

1. A

If -1 and 1 are two real roots of the polynomial function, then $f(-1)=0$ and $f(1)=0$. Thus

$f(-1)=a(-1)^3+b(-1)^2+c(-1)+d=0$ and

$f(1)=a(1)^3+b(1)^2+c(1)+d=0$.

Simplify the two equations and add them to each other.

$$\begin{array}{rl} & -a+b-c+d=0 \\ + & \underline{\ a+b+c+d=0} \\ & 2b \quad +2d=0 \text{ or } b+d=0. \end{array}$$

Also $f(0)=3$, since the graph of the polynomial passes through $(0,3)$.

$f(0)=a(0)^3+b(0)^2+c(0)+d=3$ implies $d=3$.

Substituting $d=3$ in the equation $b+d=0$ gives $b+3=0$, or $b=-3$.

2. C

If polynomial $p(x)=81x^5-121x^3-36$ is divided by $x+1$, the remainder is $p(-1)$.

$p(-1)=81(-1)^5-121(-1)^3-36=4$

The remainder is 4.

3. D

If $x-2$ is a factor for polynomial $p(x)$, then $p(2)=0$.

$p(x)=a(x^3-2x)+b(x^2-5)$

$p(2)=a(2^3-2(2))+b(2^2-5)$

$=a(8-4)+b(4-5)$

$=4a-b=0$

4. B

If $(x-a)$ is a factor of $f(x)$, then $f(a)$ must be equal to 0. Based on the table, $f(-3)=0$.

Therefore, $x+3$ must be a factor of $f(x)$.

5. 8

$x^3-8x^2+3x-24=0$	
$(x^3-8x^2)+(3x-24)=0$	Group terms.
$x^2(x-8)+3(x-8)=0$	Factor out the GCF.
$(x^2+3)(x-8)=0$	Distributive Property
$x^2+3=0$ or $x-8=0$	Solutions

Since $x^2+3=0$ does not have a real solution, $x-8=0$, or $x=8$, is the only solution that makes the equation true.

6. 3

$x^4-8x^2=9$	
$x^4-8x^2-9=0$	Make one side 0.
$(x^2-9)(x^2+1)=0$	Factor.
$(x+3)(x-3)(x^2+1)=0$	Factor.

Since $x^2+1=0$ does not have a real solution, the solutions for x are $x=-3$ and $x=3$. Since it is given that $x>0$, $x=3$ is the only solution to the equation.

Section 13-3

1. B

$$a^{-\frac{1}{2}}=\frac{1}{a^{\frac{1}{2}}}=\frac{1}{\sqrt{a}}$$

2. C

$\dfrac{1}{3-2\sqrt{2}}$	
$=\dfrac{1}{3-2\sqrt{2}}\cdot\dfrac{3+2\sqrt{2}}{3+2\sqrt{2}}$	Multiply the conjugate of of the denominator.
$=\dfrac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$	$(a-b)(a+b)=a^2-b^2$
$=\dfrac{3+2\sqrt{2}}{9-8}$	Simplify.
$=3+2\sqrt{2}$	

3. B

$(x+1)^3=-64$	
$x+1=\sqrt[3]{-64}$	Definition of cube root.
$x+1=-4$	$\sqrt[3]{-64}=(-64)^{\frac{1}{3}}=-4$
$x=-5$	Subtract 1 from each side.

4. A

$\sqrt{8}+\sqrt{18}-\sqrt{32}$

$=\sqrt{4}\sqrt{2}+\sqrt{9}\sqrt{2}-\sqrt{16}\sqrt{2}$

$=2\sqrt{2}+3\sqrt{2}-4\sqrt{2}$

$=\sqrt{2}$

5. D

$(1+\sqrt{3})(2-\sqrt{3})$

$= 2-\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ FOIL

$= 2+\sqrt{3}-3$ Combine like radicals.

$= -1+\sqrt{3}$ Simplify.

6. D

$b^{\frac{5}{3}} = b^1 \cdot b^{\frac{2}{3}} = b \cdot (b^2)^{\frac{1}{3}} = b \cdot \sqrt[3]{b^2}$

Section 13-4

1. B

$11-\sqrt{2x+3} = 8$

$11-\sqrt{2x+3}-11 = 8-11$ Subtract 11 from each side.

$-\sqrt{2x+3} = -3$ Simplify.

$(-\sqrt{2x+3})^2 = (-3)^2$ Square each side.

$2x+3 = 9$ Simplify.

$2x = 6$ Subtract 3 from each side.

$x = 3$ Divide each side by 2.

2. A

$\sqrt{-3x+4} = 7$

$(\sqrt{-3x+4})^2 = (7)^2$ Square each side.

$-3x+4 = 49$ Simplify.

$-3x = 45$ Subtract 4 from each side.

$x = -15$ Divide each side by -3.

3. B

$\sqrt{x+18} = x-2$

$(\sqrt{x+18})^2 = (x-2)^2$ Square each side.

$x+18 = x^2-4x+4$ Simplify.

$0 = x^2-5x-14$ Make one side 0.

$0 = (x-7)(x+2)$ Factor.

$0 = x-7$ or $0 = x+2$ Zero Product Property

$7 = x$ or $-2 = x$

Check each x-value in the original equation.

$\sqrt{7+18} = 7-2$ $x = 7$

$\sqrt{25} = 5$ Simplify.

$5 = 5$ True

$\sqrt{-2+18} = -2-2$ $x = -2$

$\sqrt{16} = -4$ Simplify.

$4 = -4$ False

Thus, 7 is the only solution.

4. C

$\sqrt{5x-12} = 3\sqrt{2}$

$(\sqrt{5x-12})^2 = (3\sqrt{2})^2$ Square each side.

$5x-12 = 18$ Simplify.

$5x = 30$ Add 12 to each side.

$x = 6$ Divide by 5 on each side.

5. $\frac{5}{9}$

$\sqrt{2-3x} = \frac{1}{3}a$

$\sqrt{2-3x} = \frac{1}{3}\sqrt{3}$ $a = \sqrt{3}$

$(\sqrt{2-3x})^2 = (\frac{1}{3}\sqrt{3})^2$ Square each side.

$2-3x = \frac{1}{3}$ Simplify.

$-3x = -\frac{5}{3}$ Subtract 2 from each side.

$-\frac{1}{3}(-3x) = -\frac{1}{3}(-\frac{5}{3})$ Multiply each side by $-\frac{1}{3}$.

$x = \frac{5}{9}$ Simplify.

6. 2

$\sqrt[3]{x-k} = -2$

$(\sqrt[3]{x-k})^3 = (-2)^3$ Cube each side.

$x-k = -8$ Simplify.

$x-(8-\sqrt{2}) = -8$ $k = 8-\sqrt{2}$

$x-8+\sqrt{2} = -8$ Simplify.

$x+\sqrt{2} = 0$ Add 8 to each side.

$x = -\sqrt{2}$ Subtract $\sqrt{2}$.

$(x)^2 = (-\sqrt{2})^2$ Square each side.

$x^2 = 2$ Simplify.

Section 13-5

1. B

$\sqrt{-1}-\sqrt{-4}+\sqrt{-9}$

$= i-i\sqrt{4}+i\sqrt{9}$ $i = \sqrt{-1}$

$= i-2i+3i$

$= 2i$

2. C

$\sqrt{-2} \cdot \sqrt{-8}$

$= i\sqrt{2} \cdot i\sqrt{8}$ $\qquad \sqrt{-2} = i\sqrt{2}$, $\sqrt{-8} = i\sqrt{8}$

$= i^2\sqrt{16}$

$= -4$ $\qquad i^2 = -1$

3. D

$\dfrac{3-i}{3+i}$

$= \dfrac{3-i}{3+i} \cdot \dfrac{3-i}{3-i}$ $\qquad$ Rationalize the denominator.

$= \dfrac{9-6i+i^2}{9-i^2}$ $\qquad$ FOIL

$= \dfrac{9-6i-1}{9+1}$ $\qquad i^2 = -1$

$= \dfrac{8-6i}{10}$ $\qquad$ Simplify.

$= \dfrac{4-3i}{5}$ or $\dfrac{4}{5} - \dfrac{3i}{5}$

4 . C

$\dfrac{1}{2}(5i-3) - \dfrac{1}{3}(4i+5)$

$= \dfrac{5}{2}i - \dfrac{3}{2} - \dfrac{4i}{3} - \dfrac{5}{3}$ $\qquad$ Distributive Property

$= \dfrac{15}{6}i - \dfrac{9}{6} - \dfrac{8i}{6} - \dfrac{10}{6}$ $\qquad$ 6 is the GCD.

$= \dfrac{7}{6}i - \dfrac{19}{6}$ $\qquad$ Simplify.

5. 23

$(4+i)^2 = a+bi$

$16+8i+i^2 = a+bi$ $\qquad$ FOIL

$16+8i-1 = a+bi$ $\qquad i^2 = -1$

$15+8i = a+bi$ $\qquad$ Simplify.

$15 = a$ and $8 = b$ $\qquad$ Definition of Equal Complex Numbers

Therefore, $a+b = 15+8 = 23$.

6. 2

$\dfrac{3-i}{1-2i} = \dfrac{3-i}{1-2i} \cdot \dfrac{1+2i}{1+2i} = \dfrac{3+6i-i-2i^2}{1-4i^2}$

$= \dfrac{3+6i-i+2}{1+4} = \dfrac{5+5i}{5} = 1+i = a+bi$

Therefore, $a = 1$ and $b = 1$, and $a+b = 1+1 = 2$.

Chapter 13 Practice Test

1. C

$f(x) = 2x^3 + bx^2 + 4x - 4$

$f(\frac{1}{2}) = 0$ because the graph of f intersects the x-axis at $(\frac{1}{2}, 0)$.

$f(\frac{1}{2}) = 2(\frac{1}{2})^3 + b(\frac{1}{2})^2 + 4(\frac{1}{2}) - 4 = 0$

Solving the equation for b gives $b = 7$.

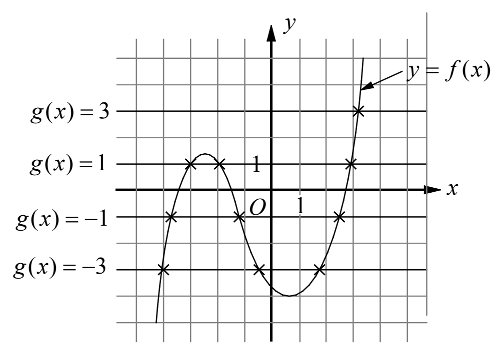
Thus $f(x) = 2x^3 + 7x^2 + 4x - 4$.

Also $k = f(-2)$, because $(-2, k)$ lies on the graph of f.

$k = f(-2) = 2(-2)^3 + 7(-2)^2 + 4(-2) - 4$

Solving the equation for k gives $k = 0$.

2. D



$g(x) = -3$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = -1$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = 1$ has 3 points of intersection with $y = f(x)$, so there are 3 real solutions.

$g(x) = 3$ has 1 point of intersection with $y = f(x)$, so there is 1 real solution.

Choice D is correct

3. B

If $x+2$ is a factor of $f(x) = -(x^3 + 3x^2) - 4(x-a)$, then $f(-2) = 0$.

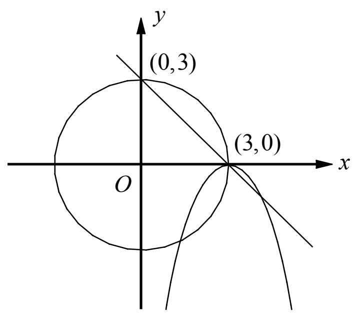
$f(-2) = -((-2)^3 + 3(-2)^2) - 4(-2-a) = 0$

$-(-8+12) + 8 + 4a = 0$

$4 + 4a = 0$

$a = -1$

4. A



The solutions to the system of equations are the points where the circle, parabola, and line all intersect. That point is $(3,0)$ and is therefore the only solution to the system.

5. C

$$\frac{(1-i)^2}{1+i}$$

$= \dfrac{1-2i+i^2}{1+i}$ FOIL the numerator.

$= \dfrac{1-2i-1}{1+i}$ $i^2=-1$

$= \dfrac{-2i}{1+i}$ Simplify.

$= \dfrac{-2i}{1+i}\cdot\dfrac{1-i}{1-i}$ Rationalize the denominator.

$= \dfrac{-2i+2i^2}{1-i^2}$ FOIL

$= \dfrac{-2i-2}{2}$ $i^2=-1$

$= -i-1$

6. B

$$a\sqrt[3]{a} = a\cdot a^{\frac{1}{3}} = a^{1+\frac{1}{3}} = a^{\frac{4}{3}}$$

7. B

$p(x) = -2x^3+4x^2-10x$

$q(x) = x^2-2x+5$

In $p(x)$, factoring out the GCF, $-2x$, yields $p(x) = -2x(x^2-2x+5) = -2x\cdot q(x)$.

Let's check each answer choice.

A) $f(x) = p(x) - \dfrac{1}{2}q(x)$

$= -2x\cdot q(x) - \dfrac{1}{2}q(x) = (-2x-\dfrac{1}{2})q(x)$

$q(x)$ is not a factor of $x-1$ and $(-2x-\dfrac{1}{2})$ is not a factor of $x-1$. $f(x)$ is not divisible by $x-1$.

B) $g(x) = -\dfrac{1}{2}p(x) - q(x)$

$= -\dfrac{1}{2}[-2x\cdot q(x)] - q(x) = (x-1)q(x)$

Since $g(x)$ is $x-1$ times $q(x)$, $g(x)$ is divisible by $x-1$.
Choices C and D are incorrect because $x-1$ is not a factor of the polynomials $h(x)$ and $k(x)$.

8. D

$\sqrt{2x+6} = x+3$

$(\sqrt{2x+6})^2 = (x+3)^2$ Square each side.

$2x+6 = x^2+6x+9$ Simplify.

$x^2+4x+3=0$ Make one side 0.

$(x+1)(x+3)=0$ Factor.

$x+1=0$ or $x+3=0$ Zero Product Property

$x=-1$ or $x=-3$

Check each x-value in the original equation.

$\sqrt{2(-1)+6} = -1+3$ $x=-1$

$\sqrt{4}=2$ Simplify.

$2=2$ True

$\sqrt{2(-3)+6} = -3+3$ $x=-3$

$0=0$ True

Thus, -1 and -3 are both solutions to the equation.

9. C

Use the remainder theorem.

$$p(\tfrac{1}{2}) = 24(\tfrac{1}{2})^3 - 36(\tfrac{1}{2})^2 + 14 = 8$$

Therefore, the remainder of polynomial $p(x) = 24x^3-36x^2+14$ divided by $x-\dfrac{1}{2}$ is 8.

10. D

If $(x-a)$ is a factor of $f(x)$, then $f(a)$ must equal to 0. Thus, if $x+2$, $x+1$ and $x-1$ are factors of f, we have $f(-2)=f(-1)=f(1)=0$.

Choice D is correct.