Answer Key

Section 11-1

1. D	2. A	3. $\frac{9}{16}$	4. 2	5. $\frac{4}{3}$

Section 11-2

1. C	2. B	3. A	4. A	5. C
6. D				

Section 11-3

1. A	2. D	3. B	4. D	5. C
6. B				

Section 11-4

1. D	2. A	3. C	4. D	5. 11
6. $\frac{9}{16}$				

Section 11-5

1. C	2. A	3. D	4. D	5. B
6. C				

Section 11-6

1. C	2. B	3. C	4. D

Chapter 11 Practice Test

1. B	2. C	3. A	4. A	5. D
6. C	7. A	8. B	9. B	

Answers and Explanations

Section 11-1

1. D

Change the given equation into the vertex form $y = a(x-h)^2 + k$, in which (h,k) is the vertex of the parabola, by completing the square.

$y = x^2 - 6x + 5$

$= x^2 - 6x + (\frac{-6}{2})^2 - (\frac{-6}{2})^2 + 5$

$= (x^2 - 6x + 9) - 9 + 5$

$= (x-3)^2 - 4$

The coordinate of the vertex can be read as $(3,-4)$.

2. A

Change the given equation into the factored form $y = (x-a)(x-b)$, in which $x = a$ and $x = b$ are the x-intercepts of the parabola. Find two numbers with a sum of -6 and a product of 5. The two numbers are -1 and -5.

$y = x^2 - 6x + 5$ can be written in the factored form $y = (x-1)(x-5)$. The x-intercepts are 1 and 5.

3. $\frac{9}{16}$

$y = a(x-h)^2$

$0 = a(4-h)^2$ $\quad$ x-intercept at $(4,0)$

Since $a \neq 0$, $4-h=0$, or $h=4$.

The graph of the parabola passes through $(0,9)$, since the y-intercept of the parabola is 9.

$9 = a(0-h)^2$ $\quad$ y-intercept at $(0,9)$

$9 = ah^2$ $\quad$ Simplify.

$9 = a(4)^2$ $\quad$ Substitute 4 for h.

$\frac{9}{16} = a$

4. 2

$y = a(x+2)^2 - 15$

$3 = a(1+2)^2 - 15$ $\quad$ $x=1$ and $y=3$

$3 = 9a - 15$

$18 = 9a$

$2 = a$

5. $\frac{4}{3}$

The x-intercepts of the graph of the equation $y = a(x-1)(x+5)$ are -5 and 1. The x-coordinate of the vertex is the average of the two x-intercepts. Therefore, $h = \frac{-5+1}{2} = -2$. The value of k is -12 because the minimum value of y is -12. So the coordinate of the vertex is $(-2,-12)$. Substitute $x=-2$ and $y=-12$ in the given equation.

$-12 = a(-2-1)(-2+5)$

$-12 = -9a$

$\frac{12}{9} = a$ or $a = \frac{4}{3}$

Section 11-2

1. C

$x^2 - 2x - 24$

Find two numbers with a sum of -2 and a product of -24. The two numbers are -6 and 4.

Therefore, $x^2 - 2x - 24 = (x-6)(x+4)$.

2. B

$x^2 - 17x + 72$

Find two numbers with a sum of -17 and a product of 72. The two numbers are -8 and -9.

Therefore, $x^2 - 17x + 72 = (x-8)(x-9)$.

3. A

$-x^2 + 5x + 84 = -(x^2 - 5x - 84)$

Find two numbers with a sum of -5 and a product of -84. The two numbers are -12 and 7.

$-x^2 + 5x + 84 = -(x^2 - 5x - 84)$

$= -(x-12)(x+7) = (12-x)(x+7)$

4. A

$3x^2 + 7x - 6$

Find two numbers with a sum of 7 and a product of $3 \cdot -6$ or -18. The two numbers are -2 and 9.

$3x^2 + 7x - 6$	
$= 3x^2 - 2x + 9x - 6$	Write $7x$ as $-2x + 9x$.
$= (3x^2 - 2x) + (9x - 6)$	Group terms.
$= x(3x-2) + 3(3x-2)$	Factor out the GCF.
$= (3x-2)(x+3)$	Distributive Property

5. C

$2x^2 + x - 15$

Find two numbers with a sum of 1 and a product of $2 \cdot -15$ or -30. The two numbers are -5 and 6.

$2x^2 + x - 15$	
$= 2x^2 - 5x + 6x - 15$	Write x as $-5x + 6x$.
$= (2x^2 - 5x) + (6x - 15)$	Group terms.
$= x(2x-5) + 3(2x-5)$	Factor out the GCF.
$= (2x-5)(x+3)$	Distributive Property

6. D

$-6x^2 + x + 2 = -(6x^2 - x - 2)$

Find two numbers with a sum of -1 and a product of $6 \cdot -2$ or -12. The two numbers are -4 and 3.

$-6x^2 + x + 2$	
$= -(6x^2 - x - 2)$	
$= -(6x^2 - 4x + 3x - 2)$	Write $-x$ as $-4x + 3x$.
$= -[(6x^2 - 4x) + (3x - 2)]$	Group terms.
$= -[2x(3x-2) + (3x-2)]$	Factor out the GCF.
$= -(3x-2)(2x+1)$	Distributive Property

Section 11-3

1. A

$3x^2 - 48$	
$= 3(x^2 - 16)$	Factor out the GCF.
$= 3((x)^2 - (4)^2)$	Write in the form $a^2 - b^2$.
$= 3(x-4)(x+4)$	Difference of Squares

2. D

$x - 6\sqrt{x} - 16$

Let $y = \sqrt{x}$, then $y^2 = x$.

$x - 6\sqrt{x} - 16$	
$= y^2 - 6y - 16$	$y = \sqrt{x}$ and $y^2 = x$
$= (y-8)(y+2)$	
$= (\sqrt{x} - 8)(\sqrt{x} + 2)$	$y = \sqrt{x}$ and $y^2 = x$

3. B

$(x-y)^2$	
$= (x-y)(x-y)$	
$= x^2 - 2xy + y^2$	
$= (x^2 + y^2) - 2xy$	
$= 10 - 2(-3) = 16$	$x^2 + y^2 = 10$ and $xy = -3$

4. D

$x^2 - y^2$	
$= (x+y)(x-y)$	
$= (10)(4)$	
$= 40$	$x + y = 10$ and $x - y = 4$

5. C

$6x^2 + 7x - 24 = 0$

$(3x+8)(2x-3) = 0$ Factor.

$3x+8=0$ or $2x-3=0$ Zero Product Property

$x = -\frac{8}{3}$ or $x = \frac{3}{2}$ Solve each equation.

Since $\frac{3}{2} > -\frac{8}{3}$, $r = \frac{3}{2}$ and $s = -\frac{8}{3}$.

$r - s = \frac{3}{2} - (-\frac{8}{3}) = \frac{9}{6} + \frac{16}{6} = \frac{25}{6}$

6. B

$x^2 - 3x = 28$

$x^2 - 3x - 28 = 0$ Make one side 0.

$(x-7)(x+4) = 0$ Factor.

$x-7=0$ or $x+4=0$ Zero Product Property

$x=7$ or $x=-4$ Solve each equation.

Therefore, $r+s = 7+(-4) = 3$.

Section 11-4

1. D

$x^2 - 10x = 75$

Add $(\frac{-10}{2})^2$ to each side.

$x^2 - 10x + (-\frac{10}{2})^2 = 75 + (-\frac{10}{2})^2$

$x^2 - 10x + 25 = 75 + 25$ Simplify.

$(x-5)^2 = 100$ Factor $x^2 - 10x + 25$.

$x - 5 = \pm 10$ Take the square root.

$x = 5 \pm 10$ Add 5 to each side.

$x = 5+10$ or $x = 5-10$ Separate the solutions.

$x = 15$ or $x = -5$ Simplify.

If $x < 0$, $x = -5$. Therefore, $x + 5 = -5 + 5 = 0$.

2. A

$x^2 - kx = 20$

Add $(\frac{-k}{2})^2$ to each side.

$x^2 - kx + (\frac{-k}{2})^2 = 20 + (\frac{-k}{2})^2$

$x^2 - kx + \frac{k^2}{4} = 20 + \frac{k^2}{4}$ Simplify.

$(x - \frac{k}{2})^2 = 20 + \frac{k^2}{4}$ Factor $x^2 - kx + \frac{k^2}{4}$.

$(6)^2 = 20 + \frac{k^2}{4}$ Substitute 6 for $x - \frac{k}{2}$.

$16 = \frac{k^2}{4}$

Solving for k gives $k = \pm 8$.

Solving the given equation $x - \frac{k}{2} = 6$ for x gives $x = 6 + \frac{k}{2}$.

If $k = 8$, $x = 6 + \frac{k}{2} = 6 + \frac{8}{2} = 10$.

If $k = -8$, $x = 6 + \frac{k}{2} = 6 + \frac{-8}{2} = 2$.

Of the answer choices, 2 is a possible value of x. Therefore, Choice A is correct.

3. C

$x^2 - \frac{k}{3}x = 5$

The equation could be solved by completing the square by adding $(\frac{1}{2} \cdot \frac{k}{3})^2$, or $\frac{k^2}{36}$, to each side. Choice C is correct.

4. D

$x^2 - rx = \frac{k^2}{4}$

Add $(\frac{-r}{2})^2$, or $\frac{r^2}{4}$, to each side.

$x^2 - rx + \frac{r^2}{4} = \frac{k^2}{4} + \frac{r^2}{4}$

$(x - \frac{r}{2})^2 = \frac{k^2 + r^2}{4}$ Factor $x^2 - rx + \frac{r^2}{4}$.

$x - \frac{r}{2} = \pm\sqrt{\frac{k^2 + r^2}{4}}$ Take the square root.

$x - \frac{r}{2} = \pm\frac{\sqrt{k^2 + r^2}}{2}$ Simplify.

$x = \frac{r}{2} \pm \frac{\sqrt{k^2 + r^2}}{2}$ Add $\frac{r}{2}$ to each side.

Choice D is correct.

5. 11

$(x-7)(x-s) = x^2 - rx + 14$

$x^2 - (s+7)x + 7s = x^2 - rx + 14$

Since the x-terms and constant terms have to be equal on both sides of the equation,

$r = s + 7$ and $7s = 14$.
Solving for s gives $s = 2$.
$r = s + 7 = 2 + 7 = 9$
Therefore, $r + s = 9 + 2 = 11$.

6. $\frac{9}{16}$

$x^2 - \frac{3}{2}x + c = (x - k)^2 \Rightarrow$

$x^2 - \frac{3}{2}x + c = x^2 - 2kx + k^2$

Since the x-terms and constant terms have to be equal on both sides of the equation,

$2k = \frac{3}{2}$ and $c = k^2$.

Solving for k gives $k = \frac{3}{4}$.

Therefore, $c = k^2 = (\frac{3}{4})^2 = \frac{9}{16}$.

Section 11-5

1. C

$(p-1)x^2 - 2x - (p+1) = 0$
Use the quadratic formula to find the solutions for x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(p-1)(-(p+1))}}{2(p-1)}$$

$$= \frac{2 \pm \sqrt{4 + 4(p-1)(p+1)}}{2(p-1)}$$

$$= \frac{2 \pm \sqrt{4 + 4p^2 - 4}}{2(p-1)}$$

$$= \frac{2 \pm \sqrt{4p^2}}{2(p-1)} = \frac{2 \pm 2p}{2(p-1)}$$

$$= \frac{2(1 \pm p)}{2(p-1)} = \frac{1 \pm p}{p-1}$$

The solutions are $\frac{1+p}{p-1}$ and $\frac{1-p}{p-1}$, or -1.

Choice C is correct.

2. A

Let r_1 and r_2 be the solutions of the quadratic equation $3x^2 + 12x - 29 = 0$.
Use the sum of roots formula.

$r_1 + r_2 = -\frac{b}{a} = -\frac{12}{3} = -4$.

3. D

$kx^2 + 6x + 4 = 0$

If the quadratic equation has exactly one solution, then $b^2 - 4ac = 0$.

$b^2 - 4ac = 6^2 - 4(k)(4) = 0 \Rightarrow 36 - 16k = 0$

$\Rightarrow k = \frac{36}{16} = \frac{9}{4}$

4. D

$y = bx - 3$ and $y = ax^2 - 7x$
Substitute $bx - 3$ for y in the quadratic equation.

$bx - 3 = ax^2 - 7x$

$ax^2 + (-7 - b)x + 3 = 0$ Make one side 0.

The system of equations will have exactly two real solutions if the discriminant of the quadratic equation is positive.
$(-7 - b)^2 - 4a(3) > 0$, or $(7 + b)^2 - 12a > 0$.
We need to check each answer choice to find out for which values of a and b the system of equations has exactly two real solutions.

A) If $a = 3$ and $b = -2$, $(7-2)^2 - 12(3) < 0$.

B) If $a = 5$ and $b = 0$, $(7+0)^2 - 12(5) < 0$.

C) If $a = 7$ and $b = 2$, $(7+2)^2 - 12(7) < 0$.

D) If $a = 9$ and $b = 4$, $(7+4)^2 - 12(9) > 0$.

Choice Dis correct.

5. B

$x^2 + 4 = -6x$

$x^2 + 6x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm 2\sqrt{5}}{2}$$

$$= -3 \pm \sqrt{5}$$

6. C

If the quadratic equation has no real solution, the discriminant, $b^2 - 4ac$, must be negative. Check each answer choice.

A) $5x^2 - 10x = 6 \Rightarrow 5x^2 - 10x - 6 = 0$
$b^2 - 4ac = (-10)^2 - 4(5)(-6) > 0$

B) $4x^2 + 8x + 4 = 0$
$b^2 - 4ac = (8)^2 - 4(4)(4) = 0$

C) $3x^2 - 5x = -3 \Rightarrow 3x^2 - 5x + 3 = 0$
$b^2 - 4ac = (-5)^2 - 4(3)(3) < 0$

Choice C is correct.

Section 11-6

1. C

Since the two x-intercepts are -4 and 2, the equation of the parabola can be written as $y = a(x+4)(x-2)$. Substitute $x = 0$ and $y = \frac{16}{3}$ in the equation, since the graph of the parabola passes through $(0, \frac{16}{3})$.

$\frac{16}{3} = a(0+4)(0-2)$

Solving the equation for a gives $a = -\frac{2}{3}$.

Thus the equation of the parabola is $y = -\frac{2}{3}(x+4)(x-2)$.

The x-coordinate of the vertex is the average of the two x-intercepts: $\frac{-4+2}{2}$, or -1.

The y-coordinate of the vertex can be found by substituting -1 for x in the equation of the parabola: $y = -\frac{2}{3}(-1+4)(-1-2) = 6$.

The line passes through $(2,0)$ and $(-1,6)$.

The slope of the line is $\frac{6-0}{-1-2} = -2$. The equation of the line in point-slope form is $y - 0 = -2(x-2)$. To find the y-intercept of the line, substitute 0 for x. $y = -2(0-2) = 4$

Choice C is correct.

2. B

$y = x^2 + x$ and $y = ax - 1$

Substitute $ax - 1$ for y in the quadratic equation.

$ax - 1 = x^2 + x$

$x^2 + (-a+1)x + 1 = 0$ Make one side 0.

If the system of equations has exactly one real solution, the discriminant $b^2 - 4ac$ must be equal to 0.

$(-a+1)^2 - 4(1)(1) = 0$	$b^2 - 4ac = 0$
$a^2 - 2a + 1 - 4 = 0$	Simplify.
$a^2 - 2a - 3 = 0$	Simplify.
$(a-3)(a+1) = 0$	Factor.
$a = 3$ or $a = -1$	Solutions

Since $a > 0$, $a = 3$.

3. C

One can find the intersection points of the two graphs by setting the two functions $f(x)$ and $g(x)$ equal to one another and then solving for x. This yields $2x^2 + 2 = -2x^2 + 18$. Adding $2x^2 - 2$ to each side of the equation gives $4x^2 = 16$. Solving for x gives $x = \pm 2$.

$f(2) = 2(2)^2 + 2 = 10$ and also $f(-2) = 10$.

The two point of intersections are $(2,10)$ and $(-2,10)$. Therefore, the value of b is 10.

4. D

$x^2 + y^2 = 14$	First equation
$x^2 - y = 2$	Second equation
$x^2 = y + 2$	Second equation solved for x^2.
$y + 2 + y^2 = 14$	Substitute $y+2$ for x^2 in first equation.
$y^2 + y - 12 = 0$	Make one side 0.
$(y+4)(y-3) = 0$	Factor.
$y = -4$ or $y = 3$	Solve for y.

Substitute -4 and 3 for y and solve for x^2.

$x^2 = y + 2 = -4 + 2 = -2$.

Since x^2 cannot be negative, $y = -4$ is not a solution.

$x^2 = y + 2 = 3 + 2 = 5$

The value of x^2 is 5.

Chapter 11 Practice Test

1. B

The x-coordinate of the vertex is the average of the x-intercepts. Thus the x-coordinate of the vertex is $x = \frac{-2+6}{2} = 2$. The vertex form of the parabola can be written as $y = a(x-2)^2 + k$.
Choices A and D are incorrect because the x-coordinate of the vertex is not 2.
Also, the parabola passes through $(0,6)$.
Check choices B and C.

B) $y = -\frac{1}{2}(x-2)^2 + 8$

$6 = -\frac{1}{2}(0-2)^2 + 8$ Correct.

C) $y = -\frac{1}{2}(x-2)^2 + 9$

$6 = -\frac{1}{2}(0-2)^2 + 9$ Not correct.

Choice B is correct.

2. C

$(x+y)^2 = 324 \Rightarrow x^2 + 2xy + y^2 = 324$
$x^2 + y^2 = 324 - 2xy$
$(x-y)^2 = 16 \Rightarrow x^2 - 2xy + y^2 = 16$
$\Rightarrow x^2 + y^2 = 16 + 2xy$
Substituting $16 + 2xy$ for $x^2 + y^2$ in the equation $x^2 + y^2 = 324 - 2xy$ yields
$16 + 2xy = 324 - 2xy$.
Solving this equation for xy yields $xy = 77$.

3. A

From the graph we read the length of AD, which is 9. Let the length of $CD = w$.
Perimeter of rectangle $ABCD$ is 38.
$2 \cdot 9 + 2w = 38 \Rightarrow 2w = 20 \Rightarrow w = 10$
Therefore, the coordinates of B are $(-1,10)$ and the coordinates of C are $(8,10)$.
The equation of the parabola can be written in vertex form as $y = a(x-3)^2$.
Now substitute 8 for x and 10 for y in the equation. $10 = a(8-3)^2$. Solving for a gives $a = \frac{10}{25} = \frac{2}{5}$. Choice A is correct.

4. A

$(ax+b)(2x-5) = 12x^2 + kx - 10$

FOIL the left side of the equation.
$2ax^2 + (-5a+2b)x - 5b = 12x^2 + kx - 10$

By the definition of equal polynomials, $2a = 12$, $-5a + 2b = k$, and $5b = 10$. Thus, $a = 6$ and $b = 2$, and $k = -5a + 2b = -5(6) + 2(2) = -26$.

5. D

$h = -\frac{1}{2}gt^2 + v_0 t + h_0$
In the equation, $g = 9.8$, initial height $h_0 = 40$, and initial speed $v_0 = 35$. Therefore, the equation of the motion is $h = -\frac{1}{2}(9.8)t^2 + 35t + 40$.
Choice D is correct.

6. C

In the quadratic equation, $y = ax^2 + bx + c$, the x-coordinate of the maximum or minimum point is at $x = -\frac{b}{2a}$.
Therefore, the object reaches its maximum height when $t = -\frac{35}{2(-4.9)} = \frac{25}{7}$.

7. A

The object reaches to its maximum height when $t = \frac{25}{7}$. So substitute $t = \frac{25}{7}$ in the equation.

$h = -4.9(\frac{25}{7})^2 + 35(\frac{25}{7}) + 40 = 102.5$

To the nearest meter, the object reaches a maximum height of 103 meters.

8. B

Height of the object is zero when the object hits the ground.

$0 = -4.9t^2 + 35t + 40$

Use quadratic formula to solve for t.

$$t = \frac{-35 \pm \sqrt{35^2 - 4(-4.9)(40)}}{2(-4.9)}$$

$$= \frac{-35 \pm \sqrt{2009}}{-9.8} \approx \frac{-35 \pm 44.82}{-9.8}$$

Solving for t gives $t \approx -1$ or $t \approx 8.1$.
Since time cannot be negative, the object hits the ground about 8 seconds after it was thrown.

9. B

When an object hits the ground, $h = 0$.
$h_0 = 150$ is given.

$0 = -16t^2 + 150$	Substitution
$16t^2 = 150$	Add $16t^2$ to each side.
$t^2 = \frac{150}{16}$	Divide each side by 16.
$t = \sqrt{\frac{150}{16}} \approx 3.06$	